HW 11 CMSC 452. Morally DUE May 6

WARNING- THIS HW IS TWO PAGES. IF YOU DO NOT DO THE PROBLEMS ON PAGE 2 YOU CANNOT ASK FOR SYMPATHY. YOU HAVE BEEN WARNED.

1. (0 points) What is your name? Write it clearly. Staple your HW. When is the Final?

2. (0 points but you really should do it) Read my nodes on Primitive recursive, etc.

3. (40 points) **Definition** A function $f(x_1, \ldots, x_n)$ is *Manish-recursive* if either:

   (a) f is primitive recursive or is Ackermann's function

   (b) f is defined by the composition of (previously defined) Manish-recursive functions, i.e. if $g_1(x_1, \ldots, x_n)$, $g_2(x_1, \ldots, x_n)$, …, $g_k(x_1, \ldots, x_n)$ are Manish-recursive and $h(x_1, \ldots, x_k)$ is Manish-recursive, then

   $$f(x_1, \ldots, x_n) = h(g_1(x_1, \ldots, x_n), \ldots, g_k(x_1, \ldots, x_n))$$

   is Manish-recursive.

   (c) f is defined by recursion of two Manish-recursive functions, i.e. if $g(x_1, \ldots, x_{n-1})$ and $h(x_1, \ldots, x_{n+1})$ are Manish-recursive then the following function is also Manish-recursive

   $$\begin{aligned} f(x_1, \ldots, x_{n-1}, 0) &= g(x_1, \ldots, x_{n-1}) \\ f(x_1, \ldots, x_{n-1}, m+1) &= h(x_1, \ldots, x_{n-1}, m, f(x_1, \ldots, x_{n-1}, m)) \end{aligned}$$

   End of Definition

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   You can assume that there is a standard list of Manish-Recursive functions

   $$f_1, f_2, f_3, \ldots.$$

(a) Define a function F from N to N such that F is computable but F is NOT Manish-recursive.

(b) Define a function G from N to N such that for all Manish-recursive f, for all but a finite number of x, $G(x) > f(x)$. It should be CLEAR that it the property is true and not rely on anything else. (For example, NOT rely on that Ackermann grows so fast.)

4. (40 points). For each of the following sets write it in terms of quantifiers and hence see where in the Arithmetic Hierarchy it is. Let $M_1, M_2, \ldots$ be a standard list of Turing Machines. Let $f_1, f_2, f_3, \ldots$ be a list of Manish-recursive functions from the prior problem.

(a) $A = \{e \mid f_e$ is 0 on all evens $\}$

(b) $B = \{e \mid f_e$ is 0 for an infinite number of inputs $\}$

(c) $C = \{e \mid f_e$ is 0 for all but a finite number of inputs$\}$

(d) $D = \{e \mid M_e$ is 0 on all evens $\}$

(e) $E = \{e \mid M_e$ is 0 for an infinite number of inputs $\}$

(f) $F = \{e \mid M_e$ is 0 for all but a finite number of inputs$\}$

5. (10 points)

(a) What was your favorite theorem in the course? Should I do it again next time I teach it? Why or why not?

(b) What was your least favorite theorem in the course? Should I do it again next time I teach it? Why or why not?

6. (10 points)

(a) What is you favorite programming Lang? Should it be used in the first programming course? Why or why not?

(b) What is you least favorite programming Lang? Should it be used in the first programming course? Why or why not?