HW 2 CMSC 452. Morally DUE Feb 14
SOLUTIONS

1. (0 points) What is your name? Write it clearly. When is the midterm? Write that clearly too. Staple your HW.

2. (10 points) (RECALL that $\#_a(w)$ is the number of a's in w, $\#_b(w)$ is ....) Show that the following set is regular by drawing a DFA for it

$$\{w : \#_a(w) \equiv 0 \pmod 2 \vee \#_b(w) \equiv 1 \pmod 3\}$$

SOLUTION TO PROBLEM 2

(Hard to draw in LaTeX so I'll describe.)

The DFA has 6 states which we label (i, j) with $0 \leq i \leq 1$ and $0 \leq j \leq 2$. State (i, j) means that w has $\#_a(w) \equiv i \pmod 2$ and $\#_b(w) \equiv j \pmod 3$.

3. (20 points) Let $n_1, n_2 \geq 3$.

   (a) Consider

$$L_1 = \{w : \#_a(w) \equiv 0 \pmod{n_1} \vee \#_b(w) \equiv 1 \pmod{n_2}\}$$

   How many states is in the smallest DFA for this L? How many of those states are accepting states?

   (Some books use 'final states' for 'accepting states'.)

   (b) Consider

$$L_2 = \{w : \#_a(w) \equiv 0 \pmod{n_1} \wedge \#_b(w) \equiv 1 \pmod{n_2}\}$$

   How many states is in the smallest DFA for this L? How many of those states are accepting states?

   (Some books use Final states for Accepting states.)

SOLUTION TO PROBLEM 3

3) Both DFA's have $n_1 n_2$ states: For each (i, j) where $0 \leq i \leq n_1 - 1$, $0 \leq j \leq n_2 - 1$ are the states. If w leads to state (i, j) then

$\#_a(w) \equiv i \pmod{n_1}$

$\#_b(w) \equiv j \pmod{n_2}$

3a) The accept states are those with $i = 0$ OR $j = 0$.

Number of ordered pairs with $i = 0$: n_2.

Number of ordered pairs with $j = 0$: n_1.

SO is the answer $n_2 + n_1$. NO- since there is ONE state that has both $i = 0$ and $j = 0$, so then we have

$$n_2 + n_1 - 1$$

3b) The ONLY accept state is $(0, 0)$ so 1 accept state.

THERE IS ONE MORE PAGE

4. (30 points) We interpret strings over $\Sigma = \{0,1,2,3,4,5,6,7,8,9\}$ as numbers in base 10. A DFA CLASSIFIER is a DFA where instead of having final states has each state is labeled, so we think of the DFA as computing a function. (If on string w you end up at state q then we think of w as being mapped to the label of q.)

   (a) IF you were to write a DFA CLASSIFIER that will, on input w, tell what w is congruent to mod 11 THEN how many states would it have. Explain. (You need not write the actual DFA classifier.)

   (b) IF you were to write a DFA CLASSIFIER that will, on input w, tell what w is congruent to mod 13 THEN how many states would it have. Explain. (You need not write the actual DFA classifier.)

   (c) Describe a procedure that does the following: Given n, find the size of a DFA classifier that will, on input w, tell what w is congruent to mod n.

SOLUTION TO PROBLEM 4

4a) Lets look at powers of 10 mod 11

$10^0 \equiv 1$

$10^1 \equiv 10$

$10^2 = 1 \equiv 1$

$\vdots$

So we will need to keep track of (1) the position mod 2 (2) the sum mod 11. So that will be 22 states.

4b) Lets look at powers of 10 mod 13

$10^0 \equiv 1$

$10^1 = 10 \equiv 10$

$10^2 \equiv 9$

$10^3 \equiv 12$

$10^4 \equiv 3$

$10^5 \equiv 4$

$10^6 \equiv 1$

So we have to keep track of the position mod 6 and the sum mod 13, so thats $13 \times 6 = 78$ states.

4c)

- Input n
- Compute $n^0 \pmod{10}$, $n^1 \pmod{10}$, ... until you find the least $i \geq 1$ with $n^i \pmod{10}$.
- Need to keep track of the position mod i and the sum mod n so its in states.

5. (40 points)

(We will talk about NDFAs for this kind of problem on Thursday, if we don't get to them on Tuesday. Hence you might not want to start on this problem until after Thurday's lecture.)

(a) (0 points) Draw a NDFA for the set $\{(A, B) \mid (\exists x)[x \in A \wedge x \in B]\}$ (YES this is the one I did in class, but wait for the next few.) How many states does it have?

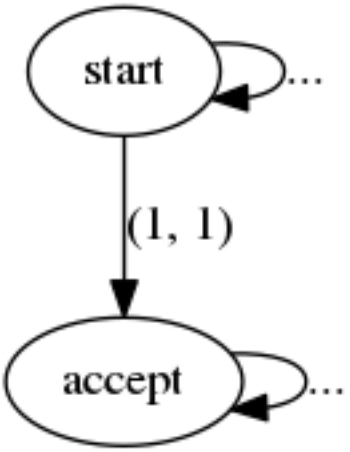
(b) (5 points) Draw a NDFA for the set $\{(A, B) \mid (\exists x)[x+1 \in A \wedge x \in B]\}$ How many states does it have?

(c) (5 points) Draw a NDFA for the set $\{(A, B) \mid (\exists x)[x+2 \in A \wedge x \in B]\}$ How many states does it have?

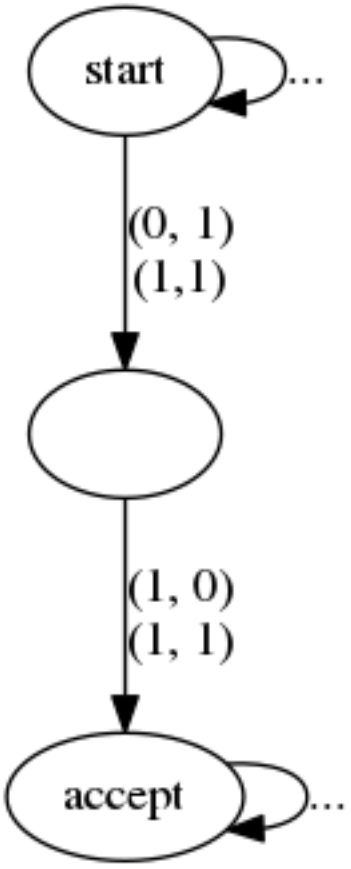
(d) (30 points) Fix $k \in \mathsf{N}$. Draw a NDFA for the set $\{(A, B) \mid (\exists x)[x+k \in A \wedge x \in B]\}$ You may use ... notation and will have to; however, make it so clear that anyone looking at your answer will be able to, given k, How many states does it have?

SOLUTION TO PROBLEM 5

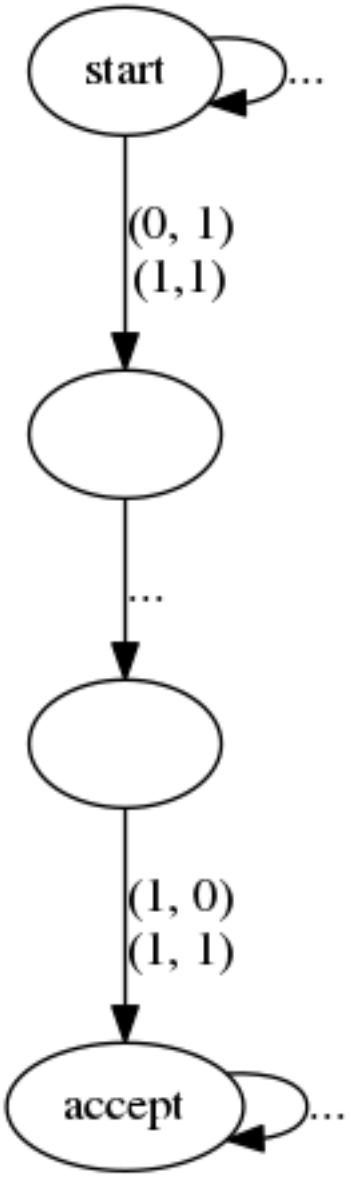
The possible transitions are $(0, 0)$ (i is not in A or B), $(1, 0)$ (i is in A but not B), $(0, 1)$, (i is in B, but not A) and $(1, 1)$ (i is in both). A "." transition is shorthand for "any transition".



(a)



(b)



(c)

(d) Will have $k + 2$ states.