

**HW 4 CMSC 452. Morally DUE Feb 28**  
**THIS HOMEWORK IS TWO PAGES**

1. (0 points) What is your name? Write it clearly. When is the midterm? Write that clearly too. Staple your HW. WHAT IS THE DAY/TIME OF THE MIDTERM? (HINT: The Midterm is March 30 IN CLASS at 11:00.)
2. (30 points)
  - (a) Write a DFA for  $\{(X, x) : x \in X \wedge x \equiv 0 \pmod{3}\}$
  - (b) Write an NFA for  $\{X : (\exists x)[x \in X \wedge x \equiv 0 \pmod{3}]\}$
3. (30 points)
  - (a) Write a DFA for:  $\{(X, x) : x \in X \wedge x \equiv 0 \pmod{4}\}$ .
  - (b) Let  $k \geq 5$ . Write a DFA for:  
 $\{(X, x) : x \in X \wedge x \equiv 0 \pmod{k}\}$ .  
(You may use  $\dots$  notation- you'll probably have to.)
4. (40 points) For this problem we consider a regular expression to use the symbols:

$\{ \quad a \quad b \quad \cup \quad e \quad \} \quad \cdot \quad *$

Each of these symbols has length 1.

- (a) Give equations for (1)  $R(i, j, 0)$  and (2)  $R(i, j, k)$  (based on  $R(*, *, k-1)$ ).
- (b) What is the longest that  $R(i, j, 0)$  can be? (We allow either  $i = j$  or  $i \neq j$  whichever one gives the longest  $R(i, j, 0)$ .)
- (c) Assume that  $(\forall i, j)[|R(i, j, k-1)| \leq L]$ . Give a bound  $L'$  such that  $(\forall i, j)[|R(i, j, k)| \leq L']$ .
- (d) Use the answer for part b to find a function  $f(k)$  that, for all  $i, j, k$ ,  $|R(i, j, k)| \leq O(f(k))$ .
- (e) Fill in the following sentence: If a regular language has a DFA with  $n$  states then it has a Reg Exp of length  $\leq O(XXX)$ .