HW 5 CMSC 452. Morally DUE Mar 7
THIS HOMEWORK IS THREE PAGES

1. (5 points) What is your name? Write it clearly. When is the midterm? Write that clearly too. Staple your HW. WHAT IS THE DAY/TIME OF THE MIDTERM? (HINT: The Midterm is March 30 IN CLASS at 11:00.)

2. (40 points)

   (a) (Use our usual convention for pairs of numbers, so the alphabet is $\{00, 01, 10, 11\}$, which we usually write vertically.)
   Write a DFA for $\{(x, y) : x = y + 3\}$. Label each state

   - A for accept,
   - R for reject, or
   - B for Bad Format.

   (NOTE - the comma after "reject" is called an *Oxford comma*.)

   (b) Write a DFA for $\{(x, y) : x \neq y + 3\}$. Label each state

   - A for accept,
   - R for reject, or
   - B for Bad Format.

3. (25 points) $A \subseteq \{0,1\}^\omega$ is D-regular if there is a DFA $M = (Q, \Sigma, \delta, s, F)$ such that:

   $x \in A \rightarrow$ if you run M on x you hit a state in F infinitely often

   $x \notin A \rightarrow$ if you run M on x you DO NOT hit a state in F infinitely often

   (a) Write a D-DFA for

   $$L_1 = \{X : X \text{ is infinite } \}.$$

   (b) Take the D-DFA you wrote for L_1 in part a. Swap the final and non-final states. Let L_1' be the language (subset of $\{0,1\}^\omega$) that your new automta accepts. Describe L_1'. Is it $\overline{L_1}$ (also called the *compliment* of L_1 or $\{0,1\}^\omega - L_1$)?

   (c) Write a D-DFA for

   $$\{X : X \text{ is infinite and } \mathsf{N} - X \text{ is infinite } \}.$$

   (d) Write an D-DFA for

   $$\{X : \text{ there is an infinite number of } x \text{ such that } x \in X \text{ and } x+1 \in X\}$$

   (e) Show this is NO D-DFA for $\{X : X \text{ is finite}\}$.

4. (35 points) A *J-automata* M is a tuple $(Q, \Sigma, \delta, s, F)$ such that

   - Q is a set of states - just like in a DFA.
   - Σ is an alphabet - just like in a DFA.
   - $\delta : Q \times \Sigma \to Q$ - just like a DFA.
   - $s \in S$, the start state - just like a DFA.
   - F is a NOT a subset of Q. F is a set of subsets of F. For example, if $Q = \{1, 2, 3, 4, 5, 6\}$ F could be $\{\{1, 2, 5\}, \{1, 5\}, \{2, 3, 6\}\}$

   Let $x \in \{0, 1\}^\omega$. We say that J-automta A *accepts* x if, when you run x through A, the set of states that are visiting infinitely often is a set in F. For example, in the above example, if the set of states visiting infinitely often was $\{1, 2, 5\}$ then ACCEPT, but if its $\{1, 2\}$ then REJECT.

   A subset of $\{0, 1\}^\omega$ which is accepted by an J-automta is called J-regular.

   (a) (0 points) How would you compliment an J-regular set?

   (b) (7 points) Show that if L is J-regular then $\overline{L}$ is J-regular.

   (c) (7 points) Give a J-automata for $\{X : \ X \text{ is infinite } \}$.

   (d) (7 points) Give a J-automata for $\{X : \ X \text{ is finite } \}$.

   (e) (7 points) Give a J-automata for $\{X : \ X \text{ is infinite and } \mathsf{N} - X \text{ is infinite}\}$.

   (f) (7 points) Let $T = \{x : x \equiv 0 \pmod 3\}$. Give a J-automata for $\{X : \ X \cap T \text{ is infinite}\}$.