## HW 6 CMSC 452. Morally DUE Mar 14 THIS HOMEWORK IS THREE PAGES

- 1. (0 points) What is your name? Write it clearly. When is the midterm? Write that clearly too. Staple your HW. WHAT IS THE DAY/TIME OF THE MIDTERM? (HINT: The Midterm is March 30 IN CLASS at 11:00.)
- 2. (40 points)

(Presburger Arithmetic convention: a string of 0's and 1's is a number in binary, and we feed it into a DFA with the lower order bits first.)

(a) (20 points) Write a DFA for

$$\{x : x \equiv 1 \pmod{3}\}$$

Note that x is in base 2.

(b) (20 points) Write a DFA for

 $\{(x,y): x \le y\}$ 

3. (30 points)

We want to write a DFA for:

$$\{(x_1, x_2, x_3, \dots, x_n, y) : x_1 + \dots + x_n = y\}$$

- (a) (5 points) If we add 3 numbers in base 2 (i.e.  $\{(x_1, x_2, x_3, y) : x_1 + x_2 + x_3 = y\}$ ), what is the largest the carry can be?
- (b) (5 points) If we add 4 numbers in base 2 what is the largest the carry can be?
- (c) (5 points) If we add 5 numbers in base 2 what is the largest the carry can be?
- (d) (5 points) If we add 6 numbers in base 2 what is the largest the carry can be?
- (e) (5 points) Make a conjecture about the biggest carry when adding *n* numbers in base 2. (For Extra Credit - NOT for a grade but for future letters from me - PROVE your conjecture.)
- (f) (5 points) If you were to write a DFA for:

 $\{(x_1, x_2, x_3, \dots, x_n, y) : x_1 + \dots x_n = y\}$ 

how many states would it have? Explain.

- 4. (30 points) For each of the following state if it is REGULAR or NOT REGULAR. Prove your statement.
  - (a)  $\{a^n: n \text{ is a square number and } n \leq 100\}$
  - (b)  $\{a^n : n \text{ is a square number and } n \ge 100\}$
  - (c)  $\{a^n a^n : n \in \mathsf{N}\}$
  - (d)  $\{w : \text{There is a subword of } w \text{ that is a palindrome}\}$
  - (e)  $\{xyx^R : x, y \in \{a, b\}^*\}$