1. (0 points) What is your name? Write it clearly. When is the midterm? Write that clearly too. Staple your HW. WHAT IS THE DAY/TIME OF THE MIDTERM? (HINT: The Midterm is March 30 IN CLASS at 11:00.)

2. (40 points)
(Presburger Arithmetic convention: a string of 0’s and 1’s is a number in binary, and we feed it into a DFA with the lower order bits first.)

(a) (20 points) Write a DFA for

\[ \{ x : x \equiv 1 \pmod{3} \} \]

Note that \( x \) is in base 2.

(b) (20 points) Write a DFA for

\[ \{ (x, y) : x \leq y \} \]
3. (30 points)

We want to write a DFA for:

\[ \{(x_1, x_2, x_3, \ldots, x_n, y) : x_1 + \ldots x_n = y \} \]

(a) (5 points) If we add 3 numbers in base 2 (i.e. \( \{ (x_1, x_2, x_3, y) : x_1 + x_2 + x_3 = y \} \)), what is the largest the carry can be?

(b) (5 points) If we add 4 numbers in base 2 what is the largest the carry can be?

(c) (5 points) If we add 5 numbers in base 2 what is the largest the carry can be?

(d) (5 points) If we add 6 numbers in base 2 what is the largest the carry can be?

(e) (5 points) Make a conjecture about the biggest carry when adding \( n \) numbers in base 2. (For Extra Credit - NOT for a grade but for future letters from me - PROVE your conjecture.)

(f) (5 points) If you were to write a DFA for:

\[ \{(x_1, x_2, x_3, \ldots, x_n, y) : x_1 + \ldots x_n = y \} \]

how many states would it have? Explain.
4. (30 points) For each of the following state if it is REGULAR or NOT REGULAR. Prove your statement.

(a) \{a^n : n \text{ is a square number and } n \leq 100\}
(b) \{a^n : n \text{ is a square number and } n \geq 100\}
(c) \{a^n a^n : n \in \mathbb{N}\}
(d) \{w : \text{ There is a subword of } w \text{ that is a palindrome}\}
(e) \{xyx^R : x, y \in \{a, b\}^*\}