

HW 7 CMSC 452. DUE DUE Mar 28

(NO late HW allowed since you have ALL of spring break AND I want to go over it on March 28 before the MIDTERM which is on March 30.)

THIS HOMEWORK IS TWO PAGES

1. (5 points) What is your name? Write it clearly. When is the midterm? Write that clearly too. Staple your HW. WHAT IS THE DAY/TIME OF THE MIDTERM? (HINT: The Midterm is March 30 IN CLASS at 11:00.)
2. (30 points) Let

$$L = \{w : \#_a(w) \text{ is a cube} \}.$$

- (a) Show that L is NOT regular using closure properties and the Aaron George Pumping Lemma
 - (b) Show that L is NOT regular using Communication Complexity.
3. (25 points) Let

$$L = \{a^p : p \text{ is prime} \}.$$

- (a) (5 points) Show that, for all natural numbers g , (' g ' for 'gap') there are two consecutive primes that are more than g apart. (You will need this for the next two problems. If you can't solve this problem then solve the next two using it, and you will get full credit on the next two, though not on this one.)
- (b) (10 points) Show that L is NOT regular using the Aaron George Pumping Lemma.
- (c) (10 points) Show that L is NOT regular using Communication Complexity.

4. (40 points) Let SQ be the set of squares, CU be the set of cubes, PR be the set of primes.

We know from class and from this HW that the following sets are not regular:

$$\{w : \#_a(w) \in SQ\}$$

$$\{w : \#_a(w) \in CU\}$$

$$\{w : \#_a(w) \in PR\}$$

THINK ABOUT: What property of SQ , CU , PR did we use in the proof?

And now the real problem: Find a property of sets PROP so that the following is true, and PROVE it:

If $B \subseteq \mathbb{N}$ has property PROP then

$$\{w : \#_a(w) \in B\}$$

is not regular. (HINT- I used Comm Complexity. There might be a proof using Pumping, I don't know.)