

## HW 9 CMSC 452. Morally Due April 18

1. (5 points) What is your name? Write it clearly. Staple the HW.
2. (20 points) Assume  $L_1 \in DTIME(T_1(n))$  and  $L_2 \in DTIME(T_2(n))$ . Show that  $L_1L_2 \in DTIME(n(T_1(n) + T_2(n)))$ . (You can write pseudo code and note how long the program runs. We ignore additive constants.)
3. (25 points) Formally define a 2-tape Turing Machine that has a head on each tape.
4. (25 points) A *Graph* is a set  $(V, E)$  where  $E \subseteq \binom{V}{2}$  (set of pairs of elements from  $V$ ). Fix  $k$ . A *k-hypergraph* is a set  $(V, E)$  where  $E \subseteq \binom{V}{k}$  (set of  $k$ -sized sets of  $V$ ). A *hypergraph* is a set  $(V, E)$  where  $E \subseteq 2^V$  (the set of all subsets of  $V$ ).

For any of these objects let  $n = |V|$  and  $m = |E|$ .

SO  $n$  is the NUMBER OF VERTICES and  $m$  is THE NUMBER OF EDGES. (Recall that  $|A|$  is the NUMBER OF ELEMENTS IN THE SET  $A$ .)

You can also assume that  $V = \{1, \dots, n\}$ .

- (a) Describe two ways to store a graph in a computer. How much space does it take as a function of  $n$  and  $m$ ?
  - (b) Describe two ways to store a  $k$ -hypergraph in a computer. How much space does it take as a function of  $n$  and  $m$ ?
  - (c) Describe two ways to store a hypergraph in a computer. How much space does it take as a function of  $n$  and  $m$ ?
5. (25 points) Let  $L \in DTIME(T(n))$ . Find a polynomial  $p$  such that  $L^* \in DTIME(p(T(n)))$ . Give the algorithm that achieves this (it can use the algorithm for  $L \in DTIME(T(n))$  and should be in pseudocode).