# HW 9 CMSC 452. Morally Due April 18 SOLUTIONS

- 1. (5 points) What is your name? Write it clearly. Staple the HW.
- 2. (20 points) Assume  $L_1 \in DTIME(T_1(n))$  and  $L_2 \in DTIME(T_2(n))$ . Show that  $L_1L_2 \in DTIME(n(T_1(n) + T_2(n)))$ . (You can write pseudo code and note how long the program runs. We ignore additive constants.)

### SOLUTION TO PROBLEM TWO

 $L_1 \in DTIME(T_1(n))$  via  $M_1, L_2 \in DTIME(T_2(n))$  via  $M_2$ .

- (a) Input(x) of length n.
- (b) Let  $x = x_1 x_2 \cdots x_n$ .
- (c) For i = 1 to n test  $x_1 \cdots x_i \in L_1$  (this takes  $\leq T_1(n)$  steps) and  $x_{i+1} \cdots x_n \in L_2$  (this takes  $\leq T_2(n)$  steps).

Notices that the key time-consuming step is the last one. It takes

$$\leq n(T_1(n) + T_2(n))$$

steps.

3. (25 points) Formally define a 2-tape Turing Machine that has a head on each tape.

### SOLUTION TO PROBLEM THREE

 $(Q,\Sigma,\delta,s,h)$ 

- (a) Q is a set of states.
- (b)  $\Sigma$  is an alphabet.
- (c)  $s \in Q$  is the start state
- (d)  $h \in Q$  is the halt state once there you are DONE.

$$\delta: Q - \{h\} \times \Sigma \times \Sigma \to Q \times \Sigma \cup \{R, L\}) \times (\Sigma \cup \{R, L\})$$

The intuition is that the the TWO heads are on TWO tapes and hence there are TWO symbols that are being seen. Having seen these two symbols the heads decide what to do. Each one moves R, moves L, or prints a symbol. They need NOT do the same thing. 4. (25 points) A *Graph* is a set (V, E) where  $E \subseteq \binom{V}{2}$  (set of pairs of elements from V). Fix k. A k-hypergraph is a set (V, E) where  $E \subseteq \binom{V}{k}$  (set of k-sized sets of V). A hypergraph is a set (V, E) where  $E \subseteq 2^V$  (the set of all subsets of V).

For any of these objects let n = |V| and m = |E|.

SO n is the NUMBER OF VERTICES and m is THE NUMBER OF EDGES. (Recall that |A| is the NUMBER OF ELEMENTS IN THE SET A.)

You an also assume that  $V = \{1, \ldots, n\}$ .

- (a) Describe two ways to store a graph in a computer. How much space does it take as a function of n and m?
- (b) Describe two ways to store a k-hypergraph in a computer. How much space does it take as a function of n and m?
- (c) Describe two ways to store a hypergraph in a computer. How much space does it take as a function of n and m?

## SOLUTION TO PROBLEM FOUR

4.a)

Use an 2-dimensional array of 0's and 1's. A[i, j] = 0 if  $(i, j) \notin E$  and 1 if  $(i, j) \in E$ . This takes  $n^2$  space.

LIST out all of the vertices and all of the edges. This takes O(n+m). 4.b) Use a k-dimensional array of 0's and 1's. Similar to graphs. This takes  $n^k$  space.

LIST out all of the vertices and all of the edges. This takes O(n+m).

4.c) Use a 1-dimensional array indexed by subsets of V. 0 if the set is in the hypergraph, 1 if its not. This takes  $2^n$  space.

LIST out all of the vertices and all of the edges. This takes O(n+m).

5. (25 points) Let  $L \in DTIME(T(n))$ . Find a polynomial p such that  $L^* \in DTIME(p(T(n)))$ . Give the algorithm that achieves this (it can use the algorithm for  $L \in DTIME(T(n))$  and should be in pseudocode).

#### SOLUTION TO PROBLEM FIVE

(We will keep an array A[i, j] with  $1 \le i \le j \le n + 1$  such that A[i, j] = 1 iff  $x_i \cdots x_{j-1} \in L^*$ .) Let  $L \in DTIME(T(n))$  via TM M. Input  $x = x_1 \cdots x_n$ . For i = 1 to n A[i, i] = 1 (since  $e \in L^*$ ). For k = 1 to nFor i = 1 to n - k j = i + kFor  $\ell = i$  to jIf  $A[i, \ell] = 1$  AND  $A[\ell, j] \in L$  then A[i, j] = 1. (We determine what A[i, j] is. Note that we know ALL A[i', j'] with j' - i' < k.)