

HW 10 CMSC 452. Morally Due April 25
THIS HW IS TWO PAGES!

1. (0 points BUT if you don't do it you'll get a 0 on the entire HW) What is your name? Write it clearly. Staple the HW.
2. (20 points) Prove that the following language is NOT context free:

$$\{a^n b^{2n} c^{3n} : n \in \mathbf{N}\}$$

3. (20 points) Let G be the following context free grammar:

$$S \rightarrow e$$

$$S \rightarrow SS$$

$$S \rightarrow bSaa$$

$$S \rightarrow aSab$$

$$S \rightarrow aSba$$

$$S \rightarrow aaSb$$

$$S \rightarrow abSa$$

$$S \rightarrow baSa$$

$$S \rightarrow aSbSa$$

- (a) (5 points) Write down 10 strings from $L(G)$. (You do NOT need to show work.)
 - (b) (5 points) Give a conjecture as to what $L(G)$ is. (Call this L')
 - (c) (10 points) Prove the easy half of your conjecture: That $L(G) \subseteq L'$.
 - (d) Extra Credit - prove the hard half.
4. (30 points)
 - (a) (15 points) Show that the set

$$\{G : G \text{ has a clique of size } 17\}$$

is in P.

(You can assume the input is an adj matrix. This goes for part b of this problem, and also the next problem.)

(b) (15 points) Let $k \in \mathbf{N}$. Show that the set

$$\{G : G \text{ has a clique of size } k\}$$

is in P.

(c) (0 points but please think about) The algorithm you got for the last problem had k in the exponent (this is still poly time since k is a constant). Do you think you can do this with \sqrt{k} in the exponent? Some smaller function of k ?

5. (30 points) RECALL: If $G = (V, E)$ is a graph then a *vertex cover of size k* is a set of vertices $U \subseteq V$ such that $(\forall (x, y) \in E)[x \in U \vee y \in U]$ (so every edge has at least one vertex in U).

(a) (15 points) Show that the set

$$\{G : G \text{ has a vertex cover of size } 17\}$$

is in P.

(b) (15 points) Let $k \in \mathbf{N}$. Show that the set

$$\{G : G \text{ has a vertex cover of size } k\}$$

is in P.

(c) (0 points but please think about) The algorithm you got for the last problem had k in the exponent (this is still poly time since k is a constant). Do you think you can do this with \sqrt{k} in the exponent? Some smaller function of k ?

6. (0 points, but Think About) Let

$$VC = \{(G, k) : G \text{ has a Vertex Cover of size } \leq k\}$$

Let

$$F_{VC}(G) = \text{the size of the smallest Vertex cover of } G$$

Show that IF $VC \in P$ then $F_{VC} \in FP$ (a function that can be computed in poly time).