THROUGHOUT THIS HW YOU MAY ASSUME:
3-COL is NP-complete
SAT is NP-complete.

1. (0 points BUT if you don’t do it you’ll get a 0 on the entire HW) What is your name? Write it clearly. Staple the HW.

2. (25 points) Let

\[ COL_k = \{ G \mid G \text{ is } k\text{-colorable} \} \]

(a) Show that \( COL_3 \leq COL_4 \).
(b) Show that \( COL_k \leq COL_{k+1} \).
(c) Show that \( COL_4 \leq COL_3 \).

3. (25 points) Let

\[ CLIQ_1 = \{ G : G \text{ has } n \text{ vertices and has a clique of size } n/3 \} \]
\[ CLIQ_2 = \{ G : G \text{ has } n \text{ vertices and has a clique of size } n/2 \} \]

(Ignore divisibility issues for 2 and 3 dividing \( n \).)

(a) Show that \( CLIQ_1 \leq CLIQ_2 \)
(b) Is either problem NP-complete? (HINT - look at the proof that \( CLIQ \) is NP-complete carefully!)

4. (25 points) A formula is in DNF FORM if it is of the form \( D_1 \lor \cdots \lor D_m \) where each \( D_i \) is the AND of literals.
\( DNF - SAT \) is the set of DNF-formulas that are SATISFIABLE.
Show either, \( DNF - SAT \) is NP-complete, or that \( DNF - SAT \) is in P.
5. (25 points) Below is an algorithm for Vertex Cover of size $k$ which has some [FILL THIS IN] in it. Your job: You guessed it!

There is a global variable, $I$, in this recursive procedure.

$VC(G, k)$

(a) Remove all isolated vertices.

(b) If there is any vertex $v$ of degree $\geq k + 1$ then $v$ MUST go into the vertex cover because [FILL THIS IN]. So $I = I \cup \{v\}$. If $|I| \geq k + 1$ then output NO and stop. Else let $G' = G - \{v\}$ and call $VC(G', k - 1)$.

(c) If there are no vertices of degree $\geq k + 1$ then EVERY vertex is of degree $\leq k$. If there is a VC of size $k$ then there are at most $k^2$ edges because [FILL THIS IN]. Hence there are at most $k^2 - 1$ vertices. By brute force you can solve this problem in time [FILL THIS IN].

For our analysis we will assume that there is an algorithm that finds vertices of degree $\geq BLAH$ and removes them in time $O(n)$. We can just use $n$ and later make the entire algorithm an $O$-of.

The run time of this algorithm is [FILL THIS IN] because [FILL THIS IN].