HW 12 CMSC 452. Morally Due May 9 THIS HW IS TWO PAGES!

THROUGHOUT THIS HW YOU MAY ASSUME: 3-COL is NP-complete SAT is NP-complete.

- 1. (0 points BUT if you don't do it you'll get a 0 on the entire HW) What is your name? Write it clearly. Staple the HW.
- 2. (25 points) Let

$$COL_k = \{G \mid G \text{ is } k \text{-colorable } \}$$

- (a) Show that $COL_3 \leq COL_4$.
- (b) Show that $COL_k \leq COL_{k+1}$.
- (c) Show that $COL_4 \leq COL_3$.
- 3. (25 points) Let

 $CLIQ1 = \{G: G \text{ has } n \text{ vertices and has a clique of size } n/3\}$

 $CLIQ2 = \{G: G \text{ has } n \text{ vertices and has a clique of size } n/2\}$

(Ignore divisibility issues for 2 and 3 dividing n.)

- (a) Show that $CLIQ1 \leq CLIQ2$
- (b) Is either problem NP-complete? (HINT look at the proof that *CLIQ* is NP-complete carefully!)
- 4. (25 points) A formula is in *DNF FORM* of it is of the form $D_1 \vee \cdots \vee D_m$ where each D_i is the AND of literals.

DNF - SAT is the set of DNF-formulas that are SATISFIABLE. Show either, DNF - SAT is NP-complete, or that DNF - SAT is in P. 5. (25 points) Below is an algorithm for Vertex Cover of size k which has some [FILL THIS IN] in it. Your job: You guessed it!

There is a global variable, I, in this recursive procedure.

VC(G,k)

- (a) Remove all isolated vertices.
- (b) If there is any vertex v of degree $\geq k + 1$ then v MUST go into the vertex cover because [FILL THIS IN]. So $I = I \cup \{v\}$. If $|I| \geq k + 1$ then output NO and stop. Else let $G' = G - \{v\}$ and call VC(G', k - 1).
- (c) If there are no vertices of degree $\geq k + 1$ then EVERY vertex is of degree $\leq k$. If there is a VC of size k then there are at most k^2 edges because [FILL THIS IN]. Hence there are at most $k^2 - 1$ vertices. By brute force you can solve this problem in time [FILL THIS IN].

For our analysis we will assume that there is an algorithm that finds vertices of degree $\geq BLAH$ and removes them in time O(n). We can just use n and later make the entire algorithm an O-of.

The run time of this algorithm is [FILL THIS IN] because [FILL THIS IN].