452 MIDTERM

Do not open this exam until you are told. Read these instructions:

1. This is a closed book exam, though ONE sheet of notes is allowed. **No calculators, or other aids are allowed**. If you have a question during the exam, please raise your hand.
2. There are 4 problems which add up to 100 points. The exam is 1 hours 15 minutes.
3. For each question show all of your work and **write legibly**. **Clearly indicate** your answers. No credit for illegible answers.
4. After the last page there is paper for scratch work. If you need extra scratch paper **after** you have filled these areas up, please raise your hand. Scratch paper must be turned in with your exam, with your name and ID number written on it, but scratch paper **will not** be graded.
5. Please write out the following statement: "*I pledge on my honor that I will not give or receive any unauthorized assistance on this examination.*"

6. Fill in the following:

NAME :
SIGNATURE :
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SECTION NUMBER :

SCORES ON PROBLEMS

Prob 1:	
Prob 2:	
Prob 3:	
Prob 4:	
TOTAL	

1. (25 points) The alphabet is $\{a, b, c\}$. Recall that $\#_a(w)$ is how many a's are in w (similar for $\#_b$ and $\#_c$). Let L be the set of strings w such that

   EITHER

   $\#_a(w) \equiv 1 \pmod 2$

   OR

   $\#_b(w) \equiv 2 \pmod 3$

   OR

   $\#_c(w) \equiv 3 \pmod 4$.

   (a) (10 points) Describe a DFA for L. How many states does it have?

   (b) (15 points) Describe an NFA for L. How many states does it have (it should be LESS THAN the number of sates in the DFA)?

   SOLUTION TO PROBLEM ONE

   1.1) The states are (i, j, k) where $0 \le i \le 1$, $0 \le j \le 2$, $0 \le k \le 3$. State (i, j, k) means that so far $\#_a(w) \equiv i \pmod 2$, $\#_b(w) \equiv j \pmod 3$, $\#_c(w) \equiv k \pmod 4$.

   $\delta((i, j, k), a) = (i + 1 \pmod 2, j, k)$

   $\delta((i, j, k), b) = (i, j + 1 \pmod 3, k)$

   $\delta((i, j, k), c) = (i, j, k + 1 \pmod 4)$

   The final states are ALL those where EITHER $i = 1$ or $j = 2$ or $k = 3$.

   There are 24 states.

   1.2) The NFA has an e-transition to three loops: one that only counts a's mod 2, one that counts b's mod 3, and one that counts c's mod 4. Each loop has final states where its supposed to in terms of congruence mod 2,3,4.

   Start state PLUS 2 PLUS 3 PLUS 4 = 10 states.

2. (25 points). A *JOSH DFA* is $(Q, \Sigma, \delta, S, F)$ where

   - Q is a set of states (as usual)
   - Σ is the alphabet (as usual)
   - $\delta : Q \times \Sigma \rightarrow Q$ (as usual)
   - $S \subseteq Q$ (THIS is what is different- S is a SET of states, not just one state)
   - $F \subseteq Q$.

   Let M be a JOSH DFA. A string x is ACCEPTED by M if THERE EXISTS a state $s \in S$ such that if you run M on x STARTING in s you end up in a state in F.

   A language L is JOSH-regular if there is a JOSH DFA M such that $x \in L$ iff $M(x)$ accepts.

   Show that if L is JOSH-regular THEN L is regular.

   SOLUTION TO PROBLEM THREE

   Let M be a JOSH DFA.

   We create a NFA that recognizes the same language.

   Have a new start state s that has e-transitions to all of the elements of S.

3. (25 points) We use the WS1S convention which is, of course, the CORRECT convention to use no matter what anyone says. Recall that, for example $(2, 5, \{0, 3, 5, 6\})$ is represented as follows:

$$\begin{array}{ccccccc} 0 & 0 & 1 & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 1 & * \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}$$

where the $*$ can be either 0 or 1.

NOW for the problem: Let

$$L_1 = \{(x, y, X) : x \in X \wedge y \notin X\}$$

$$L_2 = \{(x, y, X) : x \notin X \vee y \in X\}$$

(a) Draw a DFA for L_1. Label each state ACCEPT, REJECT, or BAD FORMAT.

(b) Draw a DFA for L_2. Label each state ACCEPT, REJECT, or BAD FORMAT.

SOLUTION OMITTED- A PAIN TO DRAW ELECTRONICALLY.

4. (25 points) Let

$$L = \{w : \#_a(w) = 2\#_b(w)\}$$

(so if there are 40 b's then there are 80 a's).

(a) (10 points) Show that L is NOT regular using the Extended Pumping Lemma.

(b) (15 points) Show that L is NOT regular using Communication Complexity.

SOLUTION TO PROBLEM 5

For both cases let n be LARGE- we decide how large later.

5.1) Let $w = a^{2n}b^n$. By AGPL there exists x, y, z

$w = xyz$

$|xy|$ is small enough so that it contains ALL a's.

Let $x = a^{n_1}$, $y = a^{n_2}$, $z = a^{n_3}b^n$

Note that $n_1 + n_2 + n_3 = 2n$ and $n_2 \neq 0$.

$xyyz = a^{n^1+2n_2+n_3}b^n$.

AH- but $n_1 + 2n_2 + n_3 = (n_1 + n_2 + n_3) + n_2 = 2n + n_2$.

so $xyyz$ has n b's but $2n + n_2$ a's, TOO MANY, so $xyyz$ is NOT in L. Contradiction.

5.2) We need only produce a fooling set for the language

$\{(x, y) : |x| = |y| = 3n/2 \wedge xy \in L\}$.

(we will assume n is even.)

of size larger than some function of n.

Note that the input will be two strings of length $3n/2$.

$(a^{3n/2}, a^{n/2}b^n)$

$(a^{3n/2-1}b^1, a^{n/2+1}b^{n-1})$

$(a^{3n/2-2}b^2, a^{n/2+2}b^{n-2})$

$\vdots$

$(a^{3n/2-i}b^i, a^{n/2+i}b^{n-i})$

$\vdots$

$(a^{3n/2-n}b^n, a^{n/2+n}b^{n-n})$

For all of these pairs the concatenation is in L.

Let $i < j$. Then we need that this:

$a^{3n/2-i}b^i \cdot a^{n/2+j}b^{n-j} \notin L$.

This string has $2n + j - i$ a's and $n + i - j$ b's.

So we need that

$2(n + i - j) \neq 2n + j - i$

$2n + 2i - 2j \neq 2n + j - i$

$2i - 2j \neq j - i$

$3i \neq 3j$

This is TRUE since $i \neq j$.

Scratch Paper