HW 1 CMSC 452. Morally DUE Feb 6
NOTE- IN PROBLEMS 2 and 3 YOU ARE ASKED TO PROVE THEOREMS
YOU MAY USE THESE THEOREMS IN PROBLEM 4

1. (0 points) What is your name? Write it clearly. Staple your HW. When
is the midterm? Where is the midterm?
AN INJECTION IS ALSO CALLED A 1-1 MAPPING.

2. (25 points) Prove that if there is an injection from \( A \) to \( B \) and an injec-
tion from \( B \) to \( A \) then there is a bijection from \( A \) to \( B \) (this is called the
Cantor-Schroder-Bernstein by some and the Schroder-Bernstein theo-
rem by others, and likely other combinations by other people. You
MAY go to the web and find a proof; however, when you write it up
put it in your own words and make sure you understand it.) You may
use this result throughout the HW.

3. (25 points)
   (a) Show there is an injection from \( \{0, 1\}^\omega \) to \( \{0, 1, 2\}^\omega \) (HINT: this
   is trivial).
   (b) Show there is an injection from \( \{0, 1, 2\}^\omega \) to \( \{0, 1\}^\omega \)
   (c) From the two above statements what can you conclude?

4. (25 points) Let \( PRIMES \) be the set of primes. Show that the set of
all functions from \( \mathbb{N} \) to \( PRIMES \) is uncountable.

5. (25 points) Let the set \( Josh \) be defined as follows:

   • If \( p \in \mathbb{Z}[x] \) and \( \alpha \) is any of the transcendental Numbers listed
     on the website of 15 awesome transcendental numbers (there is a
     pointer on the course website) then \( p(\alpha) \) is in \( Josh \).
   • If \( p \) is a polynomial with integer coefficients and \( n \in \mathbb{N}, n \geq 2, \)
     then \( p(\ln n) \) is in \( Josh \).

Is \( Josh \) countable or uncountable? Justify your answer.