HW 1 CMSC 452. Morally DUE Feb 6
SOLUTIONS
NOTE- IN PROBLEMS 2 and 3 YOU ARE ASKED TO PROVE THEOREMS YOU MAY USE THESE THEOREMS IN PROBLEM 4

1. (0 points) What is your name? Write it clearly. Staple your HW. When is the midterm? Where is the midterm?

   AN INJECTION IS ALSO CALLED A 1-1 MAPPING.

2. (25 points) Prove that if there is an injection from A to B and an injection from B to A then there is a bijection from A to B (this is called the Cantor-Schroder-Bernstein by some and the Schroder-Bernstein theorem by others, and likely other combinations by other people. You MAY go to the web and find a proof; however, when you write it up put it in your own words and make sure you understand it.) *You may use this result throughout the HW.*

3. (25 points)

   (a) Show there is an injection from $\{0,1\}^\omega$ to $\{0,1,2\}^\omega$ (HINT: this is trivial).

   (b) Show there is an injection from $\{0,1,2\}^\omega$ to $\{0,1\}^\omega$

   (c) From the two above statements what can you conclude?

   SOLUTION TO PROBLEM 3

   1) The map $f(x) = x$ is an injection.

   2) I first say how to map must the symbols 0,1,2. Map 0 to 00, 1 to 11, and 2 to 01 Now just concat. So for example

   $f(01120) = 0011110100$

   From the output you can recover the input so its an injection. For example, lets say you were told that

   $f(x) = 1101110100$ and asked what the input must have been. You know!

   We rewrite the output with spaces for clarity. It is

   11 01 11 01 00

Hence $x = 12120$.

3) There is an injection both directions, so by the previous problem we know there is a bijection from $\{0,1,2\}^{\omega}$ to $\{0,1\}^{\omega}$.

END OF SOLUTION TO PROBLEM 3

4. (25 points) Let $PRIMES$ be the set of primes. Show that the set of all functions from N to $PRIMES$ is uncountable.

   SOLUTION TO PROBLEM FOUR

   Assume, by way of contradiction, that the set of such functions is countable. So $f_1, f_2, f_3, \ldots$ is the set of all function from N to $PRIMES$. We construct a function NOT on that list

   $g(x) =$ the next prime after $f_x(x)$.

   For all i, $g(i) \neq f_i(x)$,hence g is not f_i. Hence g is not on the list.

   END OF SOLUTION TO PROBLEM FOUR

5. (25 points) Let the set *Josh* be defined as follows:

   - ($Z[x]$ is the set of polynomials in one variable x with coefficients in the Z which is the integers.) If $p(x) \in \mathsf{Z}[x]$ and α is any of the transcendental Numbers listed on the website of 15 awesome transcendental numbers (there is a pointer on the course website) then $p(\alpha)$ is in *Josh*.
   - If p is a polynomial with integer coefficients and $n \in \mathsf{N}$, $n \geq 2$, then $p(\ln n)$ is in *Josh*.

   Is *Josh* countable or uncountable? Justify your answer.

   SOLUTION TO PROBLEM FIVE

   Countable.

   $\mathsf{Z}[x]$ is countable (we showed this in class while showing that the Algebraic Numbers are countable). We list them out

$$p_1, p_2, \ldots,$$

   We define sets $A_1, A_2, \ldots$

$A_i = \{p(\alpha) : \alpha$ is one of the 15 awesome Trans Numbers $\} \cup \{p(\ln n : n \in \mathsf{N}, n \geq 2\}$

Each A_i is the union of a finite set and a countable set, hence its countable.

Josh is the union of all of the A_i's and hence is countable.

END OF SOLUTION TO PROBLEM FIVE