HW 2 CMSC 452. Morally DUE Feb 13
THIS HOMEWORK IS TWO PAGES
SOLUTIONS

1. (0 points) What is your name? Write it clearly. Staple your HW.

2. (25 points) Show that the following set is regular by drawing a DFA for it

$$\{w : n_a(w) \equiv 1 \pmod 2 \wedge n_b(w) \equiv 1 \pmod 5\}$$

SOLUTION TO PROBLEM 2

(Hard to draw in LaTeX so I'll describe.)

The DFA has 10 states which we label (i, j) with $0 \le i \le 1$ and $0 \le j \le 4$. State (i, j) means that w has $n_a(w) \equiv i \pmod 2$ and $n_b(w) \equiv j \pmod 5$.

3. (25 points) Let $n_1, n_2 \ge 5$. Let $0 \le a_1 \le n_1 - 1$. Let $0 \le a_2 \le n_2 - 1$.

   (a) Consider

$$L = \{w : n_a(w) \equiv a_1 \pmod{n_1} \wedge n_b(w) \equiv a_2 \pmod{n_2}\}$$

   Describe a DFA that accepts L. How many states does it have? How many accept states does it have?

   (Some books use Final states for Accepting states.)

   (b) If you did the last problem correctly the number of states was $n_1 n_2$. TRUE or FALSE: There exists a_1, a_2, n_1, n_2 with $n_1, n_2 \ge 5$ and $0 \le a_1 \le n_1 - 1$, $0 \le a_2 \le n_2 - 1$ such that there is a DFA for L with MUCH LESS than $n_1 n_2$ states. Justify your answer. If TRUE then show such an a_1, a_2, n_1, n_2 and the small DFA for L. If FALSE then just say FALSE- no proof needed.

SOLUTION TO PROBLEM 3

a) The DFA has $n_1 n_2$ states: For each (i, j) where $0 \le i \le n_1 - 1$, $0 \le j \le n_2 - 1$ are the states. If w leads to state (i, j) then

$n_a(w) \equiv i \pmod{n_1}$

$n_b(w) \equiv j \pmod{n_2}$

The number of states is $n_1 n_2$.

The number of accept states is 1- just the one (a_1, a_2).

b) FALSE. Intuition: The DFA HAS TO keep track of both the number-of-a's mod n_1 and the number-of-b's mod n_2. We will later in the semester make this more rigorous.

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4. (25 points)

   (a) We interpret strings over $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ as numbers in base 10. A DFA CLASSIFIER is a DFA where instead of having final states has each state is labeled, so we think of the DFA as computing a function. (If on string w you end up at state q then we think of w as being mapped to the label of q.) IF you were to write a DFA CLASSIFIER that will, on input A BASE 10 NUMBER w, tell what w is congruent to mod 15, THEN how many states would it have. Explain. (You need not write the actual DFA classifier.)

   (b) We interpret strings over $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7\}$ as numbers in base 8. IF you were to write a DFA CLASSIFIER that will, on input A BASE 8 NUMBER w, tell what w is congruent to mod 6 THEN how many states would it have. Explain. (You need not write the actual DFA classifier.)

   (c) Describe a procedure that does the following: Given b, n finds the size of a DFA classifier that will, on input A BASE b number w, tell what w is congruent to mod n.

SOLUTION TO PROBLEM 4

4a) Lets look at powers of 10 mod 15

$10^0 \equiv 1$

$10^1 \equiv 10$

$10^2 \equiv (-5)(-5) \equiv 25 \equiv 10$

$10^3 \equiv 10^2 \times 10 \equiv 10 \times 10 \equiv 10.$

$\vdots$

Hence base 10 number:

$$d_n d_{n-1} \cdots d_1 d_0 \equiv d_0 + 10d_1 + 10d_2 + \cdots + 10d_n$$

So the first digit has weight 1 but the rest have weight 10.

So we will need to keep track of 1) Is this the first digit or not (that's 2 options).

2) the weighted sum mod 15 (the weights are $(1, 10, 10, 10, \ldots)$. (That's 15 options)

So that will be 30 states.

4b) Lets look at powers of 8 mod 6

$8^0 \equiv 1$

$8^1 = 13 \equiv 2$

$8^2 \equiv 2 \times 2 \equiv 4$

$8^3 \equiv 8^2 \times 89 \times 4 \times 2 \equiv 8 \equiv 2$

$8^4 \equiv 8^3 \times 87 \times 2 \times 2 \equiv 4$

Hence base 8 number:

$$d_n d_{n-1} \cdots d_1 d_0 \equiv d_0 + 2d_1 + 4d_2 + 2d_3 + 4d_4 + \cdots$$

So the DFA needs to keep track of:

1) Is the symbol the first one, and if not then is it in an even or odd position. (That's 3 options)

2) the weighted sum mod 6 (the weights are $(1, 2, 4, 2, 4, \ldots)$. (That's 6 options)

So that will be 18 states.

c) Given b, n do compute mod n: b^0, b^1, b^2, ... until you spot a pattern (which may take a while to emerge. Say the pattern is

$p_0, \ldots, p_L, (q_0, \ldots, q_{M-1})^*$

The DFA will need to keep track of:

1) Is the symbol the first, second, ..., Lth OR is it beyond that but then what is it mod M. That's $L + M$ options.

2) Keep track of the sum mod n.

So this takes $(L + M)n$ states.

5. (25 points) Show that if L is regular than L^* is regular. (Take an DFA for L and use it to create an NDFA for L^*. Note that the empty string is in L^*.)