1. (0 points) What is your name? Write it clearly. What day is the midterm? Staple your HW.

2. In this problem the alphabet is \{a,b\}.
   
   (a) (20 points) Write a regular expression for
   \[ \{ w : n_a(w) \equiv 0 \pmod{3} \} \]
   
   (b) (20 points) Write a regular expression for
   \[ \{ w : n_a(w) \equiv 1 \pmod{3} \} \]

3. (a) (10 points) Write a DFA for the language \(\{a^i : i \neq 1000\}\) (you may use ...) How many states does it have?
   
   (b) (10 points) Let \(n \in N\). Think of \(n\) as large. Write a DFA for the language \(\{a^i : i \neq n\}\) (you may use ...) How many states does it have (this will be a function of \(n\)).
   
   (c) (0 points but please think about – Please do so by the REAL day its due, March 6 so we can discuss in class) The answer to the last part was roughly \(n\) (for example, it might be \(n + 1\)). Is the following true or false: Any NFA for \(L\) requires around \(n\) states. Try to prove or disprove.

**TURN THE PAGE**
4. In this problem we go through a VERY simple case of going from a DFA to a regular expressio. DO NOT CHEAT- follow the construction. The alphabet is \( \{a, b\} \).

(a) (10 points) Write an NFA for the language

\[
L = \{w : a \text{ is the first letter of } w \}
\]

that has only two states. Label the two states 1 and 2 where 1 is the start state and 2 is the other state (which is the only final state).

(b) (27 points) Compute, in order, and using the algorithm show in class. Show all steps.

\[
\begin{align*}
R(1, 1, 0) \\
R(1, 2, 0) \\
R(2, 1, 0) \\
R(2, 2, 0) \\
R(1, 1, 1) \\
R(1, 2, 1) \\
R(2, 1, 1) \\
R(2, 2, 1) \\
R(1, 2, 2) \text{ (this is the only one I need)}
\end{align*}
\]

SHORT CUTS YOU CAN USE (You can use other ones also but be careful)

For any reg exp \( \alpha \), \( \emptyset \cdot \alpha = \emptyset \).

If \( \sigma \in \{a, b\} \) then \( (b \cup e)^* = b^* \).

\( e^* = e \)

For any regular expression \( \alpha \), \( e\alpha = \alpha \) and \( \alpha e = \alpha \).

For any \( \sigma \in \Sigma \), \( a \cup a = a \).

(c) (3 points) From your work on part 1 write down a regular expression for \( L \). (NOTE- it should be longer than the obvious reg exp for \( L \) which is \( a(a \cup b)^* \).)