

## HW 10 CMSC 452. Morally Due April 17

- (5 points) What is your name? Write it clearly. Staple the HW.
- (30 points) Show that the following problem is in NP:  
All  $(A, b)$  where
  - $A$  is an  $n \times n$  matrix of integers.
  - $b$  is a vector of  $n$  integers
  - There exists a vector  $x$  of integers between -5 and 5 such that  $Ax = b$
  - (Think about- not to be handed in) If we allow  $x$  to be a rational then is the problem still in NP? If we allow  $x$  to be any integer (so remove the bounds -5 to 5) then is the problem in NP?
- (30 points) If  $G = (V, E)$  is a graph then  $X \subseteq V$  is a *Vertex Cover* if for every  $e \in E$ , there is a  $v \in X$  that is the endpoint of  $e$ .

Show that the following problem is in P:

$$\{G : G \text{ has a vertex cover of size } 17\}$$

- (40 points) Describe an NFA with  $\leq 500$  states (it will actually be far less than this) for the set

$$\{a^y : (y \neq 999) \wedge (y \neq 1000)\}$$

Prove that it works by showing that if  $a^y$  is rejected then  $y = 999$  or  $y = 1000$ .

HINT- For the big loop use 32 and 33. You may USE the fact that

- For all  $n \geq 992$  there exists  $x, y \in N$  such that  $n = 32x + 33y$ .
- There does not exist  $x, y \in N$  such that  $991 = 32x + 33y$ .