HW 12 CMSC 452. Morally Due May 1
HW IS TWO PAGES!

1. (5 points) What is your name? Write it clearly. Staple the HW.

2. (20 points) Let $A \in NP$. Let p_1, p_2 and B be such that:

$$A = \{x : (\exists y)[|y| = p_1(|x|) \wedge (x, y) \in B\}$$

   where B is in $DTIME(p_2(n))$. B is computed by a Turing machine that has 6 symbols in the alphabet and 10 states.

   Let x be an string of length n. Using the Cook-Levin Theorem, I can come up with a FORMULA ϕ such that

   $x \in A$ iff $\phi \in SAT$.

   How many variables does ϕ have?

3. (20 points) Write down the part of the Cook-Levin Formula that corresponds to the transition:

   $\delta(q, a) = (p, b)$

   i.e. the head does not move and prints a 'b'

4. (30 points) Show that if $A \leq B$ and $B \leq C$ then $A \leq C$.

5. (30 points) Let

$COL_2 = \{G : G$ is 2-colorable $\}$

$COL_3 = \{G : G$ is 3-colorable $\}$

$COL_4 = \{G : G$ is 4-colorable $\}$

$COL_5 = \{G : G$ is 5-colorable $\}$

$PCOL_2 = \{G : G$ is Planar and 2-colorable $\}$

$PCOL_3 = \{G : G$ is Planar and 3-colorable $\}$

$PCOL_4 = \{G : G$ is Planar and 4-colorable $\}$

$PCOL_5 = \{G : G$ is Planar and 5-colorable $\}$

For each statement below you must answer TRUE or FALSE and *explain why.* You may assume the following: (1) $P \neq NP$, (2) COL_3 is NP-complete, (3) $PCOL_3$ is NP-complete, (4) SAT is NP-complete, (5) 3-SAT is NP-complete, (6) every planar graph is 4-colorable, (7) if $A \leq B$ and $B \leq C$ then $A \leq C$.

As usual, $A \leq B$ means that there is a function f computable in poly time such that, for all x

$$x \in A \text{ iff } f(x) \in B$$

(a) $COL_2 \leq COL_3$

(b) $COL_3 \leq COL_2$

(c) $COL_3 \leq COL_4$

(d) $COL_4 \leq COL_3$

(e) $PCOL_3 \leq COL_4$

(f) $COL_3 \leq PCOL_4$