OPTIONAL PROJECT TO AVOID GETTING A D

THIS DOCUMENT IS TWO PAGES LONG

If you do this project AND end up with an F or D in the course then I will grade it and MAY use your grade to BUMP your grade up (from an F to a D-, from a D to a C-). Throughout this document “prove” means “give a construction and discuss why it works.” What you hand in must be TYPED or VERY GOOD HANDWRITING.

DUE the LAST DAY of class. Absolute Deadline.

HINT: START early. Feel free to get help from me or the TA.

CONVENTION; You can’t say ‘by theorem BLAH’ For example, if I as you to show that

If $L$ is regular than $LL$ is regular

You CANNOT say

Because regular langs are closed under concatenation

1. (0 points but you have to answer) What is your name? Write it clearly.

2. Let $L$ be regular. Prove or Disprove or state that it is unknown to science. (You may use the equivalence of DFA,’s NFA’s, and Regular Expressions).
   (a) $LL$ is regular.
   (b) $L^*$ is regular.

3. Let $L$ be in P. Prove or Disprove or state that it is unknown to science.
   (a) $LL$ is in P.
   (b) $L^*$ is in P.

4. Let $L$ be in NP. Prove or Disprove or state that it is unknown to science.
   (a) $LL$ is in NP.
   (b) $L^*$ is in NP.

THERE IS A SECOND PAGE
5. Let $L$ be decidable. Prove or Disprove or state that it is unknown to science.

   (a) $LL$ is decidable.
   (b) $L^*$ is decidable.

6. For this problem you may assume regular languages are closed under UNION, INTERSECTION, COMPLEMENTATION, and PROJECTION. Describe carefully an ALgorithm that will, on input a SENTENCE $\phi$ in WS1S, output (1) TRUE if $\phi$ is TRUE, and (2) FALSE if $\phi$ is FALSE. (Note that since $\phi$ is a sentence it is either true or false.)

7. (a) Describe the reduction of SAT to IND SET. That is, describe how you would take a formula $\phi$ (we can assume its in CNF form) and from it get a graph $G$ and a number $k$ such that $\phi \in SAT$ iff $(G, k) \in IND SET$

   (b) Use the answer to part 1 to find a graph $G$ and a number $k$ such that

   $$(x_1 \lor x_2) \land (\neg x_1 \lor x_3) \land (x_1 \lor x_2 \lor x_3) \in SAT$$

   iff

   $$(G, k) \in IND SET.$$