Homework 4 Morally Due Feb 26
THIS HOMEWORK IS TWO PAGES LONG!!!!!!!!!!!!!!!

1. (40 points) Recall that a B-NFA is an NFA where we say that an
INFINITE string is accepted if there is SOME way to process it where
it hits a final state infinitely often. Give an algorithm for the following:
given a B-NFA $M$, determine if there exists an infinite string that it
accepts.

SOLUTION TO PROBLEM ONE
We just sketch this.

(a) Input $M = (Q, \Sigma, \delta, s, F)$
(b) For all $f \in F$ determine: (1) is there a path from $s$ to $f$ (can be
all $e$), and (2) is there a path from $f$ back to $f$ (can’t be all $e$).
(c) If there is some $f$ such that the answer to (1) and (2) is YES then
output YES. If not then output NO.

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2. (30 points) The alphabet is \( \{a, b\} \). Give a B-NFA for the following languages.

In this problem note that \( \{a, b\}^\omega \) means the set of INFINITE strings of 'a's and 'b's. The superscript is an \( \omega \), not a \( w \).

(a) (15 points)

\( \{w \in \{a, b\}^\omega \mid \text{w has an infinite number of } a's \} \)

(b) (15 points)

\( \{w \in \{a, b\}^\omega \mid \text{w has a finite number of } a's \} \)

(c) (0 points) Think about: For the above languages ponder if they could be done by a B-DFA which is a DFA where we say an infinite string accepts if it hits some final state infinitely often.

**SOLUTION TO PROBLEM TWO**

Omitted. REMIND ME TO DO IN CLASS

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3. (30 points) The alphabet is \{a, b\}. Recall that \(n_a(w)\) is the number of a’s in \(w\).

(a) (10 points) Give a regular expression for

\[
\{ w | n_a(w) \equiv 0 \pmod{3} \}
\]

(b) (10 points) Give a regular expression for

\[
\{ w | n_a(w) \equiv 1 \pmod{3} \}
\]

(c) (10 points) For all \(x, y\) with \(0 < x < y\), give a regular expression for

\[
\{ w | n_a(w) \equiv x \pmod{y} \}
\]

**SOLUTION TO PROBLEM THREE**

a)

\[ b^*(b^*ab^*ab^*)^* \]

b)

\[ b^*ab^*(b^*ab^*ab^*)^* \]

c) For each \(w\), let \(\alpha_w\) be \(b^*ab^*a \cdots b^*ab^*\) where there are \(w\) a’s. Then the solution is

\[ \alpha_x(\alpha_y)^* \]