**452 FINAL-Spring 2024**

1. (20 points-5 points for each part) Let the alphabet be $\Sigma = \{a, b\}$. Let $n \in \mathsf{N}$ and $n \geq 1000$.

   Recall that $\#_a(w)$ is the number of $a$'s in $w$.

   Recall that $\#_b(w)$ is the number of $b$'s in $w$. Let

$$L = \{w : \#_a(w) \equiv 0 \pmod{n} \text{ OR } \#_b(w) \equiv 0 \pmod{n+1}\}.$$

   Note that $L$ is regular.

   (a) Give either the transition table OR draw the DFA for $L$. You may use DOT DOT DOT. Try to use as few states as possible.

   (b) How many states does the DFA in Part 1 have?

   (c) Give either the transition table OR draw the NFA for $L$. You may use DOT DOT DOT. Try to use as few states as possible.

   (d) How many states does the NFA in Part 3 have? (It should be smaller than the number of states in the DFA.)

2. (20 points) In this problem we use the WS1S conventions.

   (a) (10 points) Give a DFA for

   $$\{(x,y) : x+2=y\}.$$

   How many states does your DFA have?

   (All states are labelled A for accept or R for reject or S for stupid.)

   comments.

   (b) (10 points) Let $n \in \mathsf{N}$. Consider the language

   $$\{(x,y) : x+n=y\}.$$

   How many states would a DFA for this language have? How may are ACCEPT? How many are REJECT? How many are STUPID? (You do NOT have to draw the DFA.)

3. (20 points) Show that the following set is Diophantine by giving the relevant polynomial.

$$\{x : x \text{ is a square and } x \not\equiv 3 \pmod 4\}.$$

4. (20 points) For this problem you will assume that $P \neq NP$.

   Recall the following abbreviations.

   - REG be the set of Regular Languages.
   - P be the set of problems in polynomial time.
   - NP be ... YOU KNOW WHAT THAT IS.
   - DEC be the set of decidable sets
   - $\Sigma_1$ be the set of $\Sigma_1$ sets.

   Recall that (assuming $P \neq NP$)

$$\text{REG} \subset \text{P} \subset \text{NP} \subset \text{DEC} \subset \Sigma_1.$$

   For each of the sets on the next page say where they are.

   For example: You might say that a set is P. However, you have to say the lowest set its in. That is, if a set is in REG and you say P, then that is wrong.

   **NO explanation required.**

   **Important** 4 points for a right answer but $-2$ for a wrong answer. **For our own benefit do not guess!** I really mean it! **For your own benefit Do not guess!**

   **Note** Mercy rule: If you earn a negative score on this problem you will get a 0 on this problem.

(a) $\mathrm{CLIQ}_{1000}$, the set of graphs that have a clique of size 1000. (Recall that a Clique of a graph $G = (V, E)$ is a set $U \subseteq V$ such that ever $x, y \in U$, $(x, y) \in E$.

(b) $\mathrm{DNF}_{1000}$, satisfiable for formulas in DNF form, $D_1 \vee \cdots \vee D_m$ where each $D_i$ is a $\wedge$ of exactly 1000 variables.

(c) The set of DFA's $M$ such that $M$ accepts at least one string.

(d) The set of CFG's $G$ in Chomsky Normal Form with alphabet $\Sigma = \{a, b\}$ such that $G$ generates at least one string in $\{a, b\}^*$.

(e) $\{a^{2^n} : n \in \mathsf{N}\}$ (All strings of $a$'s that are a power-of-two in length.)

5. For this problem $\Sigma = \{0,1\}^*$.

   Let $b, c \in \mathsf{N}$. $X$ is a *(b, c)-Clyde Set* if the following happens.

   - There exists $B \subseteq \Sigma^* \times \Sigma^*$ such that $(x, y) \in B$ can be computed in time $|x|^b$.
   - $X = \{x : (\exists y, |y| = \lfloor c \log_2(|x|) \rfloor)[(x, y) \in B]\}$.

   Show that if $X$ is a $(b, c)$-Clyde Set then $X \in \mathrm{P}$. Give the time bound for your poly time algorithm for $X$ in terms of $c$ and $b$.

**Scratch Paper**