A Cautionary Tale and Help with HW06, Problem 2

1 Cautionary Tale

I was preparing the solution slides for HW 6 problem 2 and I realized I needed a CFG for

$$L = \{ w \colon \#_a(w) = \#_b(w) \}.$$

There is one in the slides (which you can use and is also below) but I was curious what CLAUDE (a ChatGPT) would do with this question.

I asked CLAUDE for a CFG for

$$L = \{ w \colon \#_a(w) = \#_b(w) \}.$$

Claude first produced the CFG:

 $S \to aSb \mid bSa \mid e$

I then typed into Claude

Your grammar does not produce abba.

It replied

You are correct. I will fix this.

It kept giving me answers that did not work and I kept giving it counterexamples.

Finally it gave me a grammar that works (its the one in my CFL slides). $S \rightarrow aSb \mid bSa \mid SS \mid e$

Caution If you use ChatGPT then really make sure it is correct! See next page for advice on hw06 problem 2.

2 Advice on HW06 Problem 2

For HW06 Problem 2 I found it was helpful to use the following set of rules $E \rightarrow aEb \mid bEa \mid EE \mid e$

And the knowledge that

$$L(E) = \{ w \colon \#_a(w) = \#_b(w) \}.$$

(You DO NOT need to prove this. The proof is on the first CFL slides but I skipped in then as its messy.)

THEN use the following: if $\#_a(w) < \#_b(w)$ then w must be of the form $x_1x_2x_3\cdots x_L$ where

 x_1, x_3, \ldots are all in b^* and one of them has a nonzero number of b's

 x_2, x_4, \ldots all have an equal number of a's and b's.

NOTE- there are other ways to obtain the grammar you want, but make sure they work!