HW 12 CMSC 452
Morally Due TUES April 29 11:00AM
Dead-Cat Due THU May 1 11:00AM

1. (40 points) Recall that a Turing Machine had a δ function

   $\delta\colon Q - \{h\} \times \Sigma \to Q \times (\Sigma \cup \{R, L\})$.

   A Leo-Turing-Machine has a δ function

   $\delta\colon Q - \{h\} \times \Sigma \to Q \times \{L, R, S\} \times \Sigma$.

   The interpret is that L is Left, R is Right, S is STAY PUT. The last Σ is for writing a character. So for Leo-Turing-Machines each step is a move AND a write (unlike our Turing Machines where each step s a move OR a write).

   For example:

   $$\delta(q, a) = (p, R, b)$$

   means that if the machine is in state q and the head is looking at an a then the machine will

   - Write a b, then
   - move R.

   We want to prove the Cook-Levin Theorem for Leo-Turing machines.

   Give the part of the formula that corresponds to

   $\delta(q, a) = (p, R, b)$.

   We'll help you begin: the formula begins

   $$(z_{i,j,\sigma_1} \wedge z_{i,j+1,(a,q)} \wedge z_{i,j+2,\sigma_2}) \to$$

   YOU FILL IN THE REST.

2. (40 points) Let

   A *Hamiltonian Path* is a path in a graph that hits every vertex exactly once.

   A *Hamiltonian Cycle* is a cycle in a graph that hits every vertex exactly once.

   A *Hamiltonian Musical* is a a musical about either *Alexander Hamilton* or *William Rowan Hamilton* (I know which one I'd want to see).

   - HAMPATH =
     $\{(G, a, b)$: G has a Ham Path that starts at a and ends at b $\}$.
   - HAMCYCLE =
     $\{G$: G has a Hamiltonian Cycle$\}$
   - FHAMCYCLE is the function that, on input a graph G, either outputs

     NO there is no Ham Cycle, or

     YES, there is one, and HERE IT IS: (then outputs a Ham cycle).
     (There may be many Ham cycles, this function outputs one of them.)

   (a) (20 points) Show that HAMPATH $\leq$ HAMCYCLE.

   (b) (10 points) Is the following problem known to be in P? NP-complete?

   $\{(G, a, b)$:

   G has a Ham Path that starts at a and ends at b AND

   every vertex in G has degree $\leq 2\}$.

   If you think it is in P then give the algorithm. If you think it is NPC then give a reference (which can be on the web).

   (c) (10 points) Is the following problem known to be in P? NP-complete?

   $\{(G, a, b)$:

   G has a Ham Path that starts at a and ends at b AND

   every vertex in G has degree $\leq 3\}$.

   If you think it is in P then give the algorithm. If you think it is NPC then give a reference (which can be on the web).

3. (20 points) (You are encouraged to go to the web for this one)

   Recall that in class we showed that (or will show that, depending on when I do it) FACTORING is probably NOT NP-complete.

   There ARE problems whose input is a SET of natural numbers that IS NP-complete.

   FIND ONE on the web and state it.