HW 13 CMSC 452
Morally Due TUES May 6 11:00AM
Dead-Cat Due THU May 8 11:00AM

1. (40 points)

   Definition Let $\Sigma = \{a, b\}$. Note that $\$ \notin \Sigma$. We will be using \$ as a separator so we can have a set of ordered pairs. Let $B \subseteq \Sigma^*\$\Sigma^*$. (For example $aabba\$bba$ could be in B but $aabb$ cannot.) We define

   $$\text{JAVIER}(B) = \{x \colon (\exists y)[|y| = 2^{2^{|x|}} \text{ AND } x\$y \in B\}.$$

   Show that if B is decidable then JAVIER(B) is decidable.

   (Here is how the proof begins: Assume you have a program M for B. Your program for JAVIER(B) can use M as a subroutine. You may use psuedocode. You DO NOT have to deal with Turing Machines at all.)

2. (30 points) Let $M_0, M_1, \ldots,$ be the set of all Turing Machines (we assume $\Sigma = \{a, b, \$, Y, N, , \#\}$ but this does not matter). You may use Google or ChatGPT or your mothers supercomputer, but you should not need any of those. We are only saying this since you might ask.

   (a) (15 points) Give a set of the form
   $\{e : M_e$ BLAH BLAH$\}$
   that is DECIDABLE and was NOT on the April 29 slides.

   (b) (15 points) Give a set of the form
   $\{e : M_e$ BLAH BLAH BLAH$\}$
   that is UNDECIDABLE and was NOT on the April 29 slides.

3. (30 points) Let A be the set of natural numbers x such that

- x is a cube, AND
- Either $x \equiv 1 \pmod 5$ OR $x \equiv 2 \mod 8$.

Show a polynomial $p \in \mathbb{Z}[y_1, \ldots, y_n]$ (I am not telling you n) such that

$$A = \{x \colon (\exists y_1, \ldots, y_n)[x \geq 0 \wedge p(y_1, \ldots, y_n, x) = 0]\}.$$