BILL START RECORDING

Two Funny Math Things I Saw Recently

Exposition by William Gasarch—U of MD

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3) If **Rolling Stone** picks **Like a Rolling Stone** as the greatest rock song of all time that might cause an infinite loop.

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Make of that what you will.

Problem 1

$$\mathrm{JAVIER}(B) = \{x : (\exists y)[|y| = 2^{2^{|x|}} \text{ AND } x\$y \in B]\}.$$

Show that if B is decidable then $\mathrm{JAVIER}(B)$ is decidable.

Problem 1 Answer

B is decidable via TM M.
Here is an algorithm for

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3) (If you got here then $(\forall y, |y| = 2^{2^{|x|}}[x\$y \notin B]$ Output NO.

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Show a polynomial $p \in \mathrm{Z}[y_1, \ldots, y_n]$ (I am not telling you n) such that

$$A = \{x : (\exists y_1, \ldots, y_n)[x \geq 0 \wedge p(y_1, \ldots, y_n, x) = 0]\}.$$

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$x \equiv 1 \pmod 5$ OR $x \equiv 2 \mod 8$:

$(\exists y_2, y_3)[x \geq 0 \wedge (x - 5y_2 - 1)(x - 8y_3 - 2) = 0]$

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x is a cube AND ($x \equiv 1 \pmod 5$ OR $x \equiv 2 \mod 8$):

$(\exists y_1, y_2, y_3)[x \geq 0 \wedge (x - y_1^3)^2 + ((x - 5y_2 - 1)(x - 8y_3 - 2))^2 = 0]$