BILL AND NATHAN START RECORDING

Context Sensitive Languages

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5) Languages that are CSL but not CFL.

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One of the motivations for CFL's and CSL's is an attempt to model human language.
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1. While human language is far more complicated than CFL or CSL; the Mathematical tools these grammars supply were a helpful starting point.
2. Computer languages are far easier to understand since we make them ourselves; hence, CFLs and (to a lesser extent) CSL's were useful within Computer Science.

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$AB \to BA$ (Note- We allow two nonterminals on the LHS.)
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3) Context-Sensitive means can replace (say) A by (say) α AND look at what is around A. We actually allow more than that.

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Don't know so won't prove. Don't care so no extra credit for it.

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In case that link goes away (plausible) and you are really eager to see the CSL (less plausible) next slide has the CSG for it (not quite).

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$AD \rightarrow DaA$
$aD \rightarrow Da$
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$HA \rightarrow AI$
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(Last four rules not allowed in a CSG but this can be dealt with.)

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So $A \Rightarrow aabb$

Example of a Lang that is NOT a CSL

We'll come back to this later.

CLOSURE PROPERTIES

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The proof that LBA-recognizers and CSG-generators are equivalent is messy so we won't be doing it. We won't deal with LBA's in this course at all.

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Hence it is easy to show that $\{a^{n^2} : n \in N\}$ and many other languages are CSL's without using CSG's.

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Open question Some variants of Chess and Go **might be** provably not CSL.

Comparing Reg, CFL, CSL

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Lang	Rcg	Gen	$\cup$	$\cap$	$\cdot$	*	comp	PL
Reg	DFA	Rgx	Y-E	Y-E	Y-E	Y-E	Y-E	Y
CFL	PDA	CFG	Y-E	N-E	Y-E	Y-E	N-E	Y
CSL	LBA	CSG	Y-E	Y-H	Y-E	Y-E	Y-H	N

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3. Proving sets are not Context-Sensitive is **Hard**.