Review for CMSC 452 Final

Deterministic Finite Automata (DFA)

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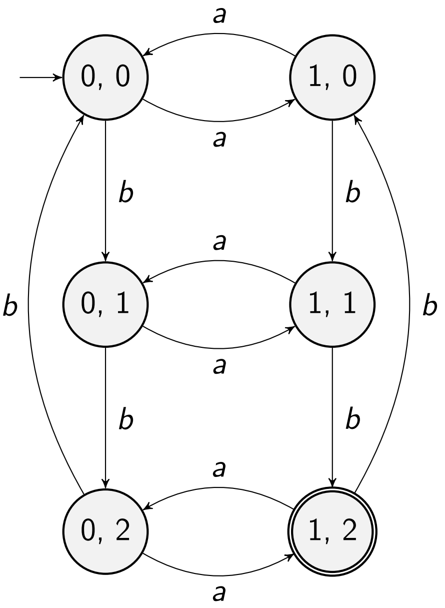

Nondeterministic Finite Automata (NFA)

NFA's Intuitively

1. An NFA is a DFA that can guess.
2. NFAs do not really exist.
3. Good for $\cup$ since can guess which one.
4. An NFA accepts iff SOME guess accepts.

Every NFA-lang a DFA-lang!

Thm If L is accepted by an NFA then L is accepted by a DFA.
Pf Sketch L is accepted by NFA $(Q, \Sigma, \Delta, s, F)$ where

1. Get rid of e-transitions using reachability.
2. Get rid of non-determinism by using power sets. Possibly 2^n blowup.

Regular Expressions

Examples

1. $b^*(ab^*ab^*)^*ab^*$
2. $b^*(ab^*ab^*ab^*)^*$
3. $(b^*(ab^*ab^*)^*ab^*) \cup (b^*(ab^*ab^*ab^*)^*)$

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REGEX $\subseteq$ NFA: Induction on formation of regex. Linear.

Closure Properties

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Property	DFA	NFA	regex
$L_1 \cup L_2$	Prod	*e*-trans	Def
$L_1 \cap L_2$	Prod	Prod	X
$\overline{L}$	Swap	X	X
$L_1 \cdot L_2$	X	*e*-trans	Def
L^*	X	*e*-trans	Def

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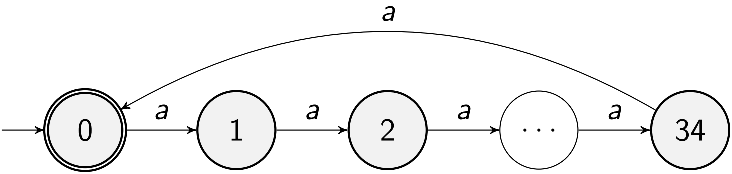
$L_1 \cup L_2$	$n_1 n_2$	$n_1 + n_2 + 1$	$\ell_1 + \ell_2$
$L_1 \cap L_2$	$n_1 n_2$	$n_1 n_2$	X
$L_1 \cdot L_2$	X	$n_1 + n_2$	$\ell_1 + \ell_2$
$\overline{L}$	n	X	X
L^*	X	$n + 1$	$\ell + 1$

Number of States for DFAs and NFAs

Minimal DFA for $L_1 = \{a^i : i \equiv 0 \pmod{35}\}$



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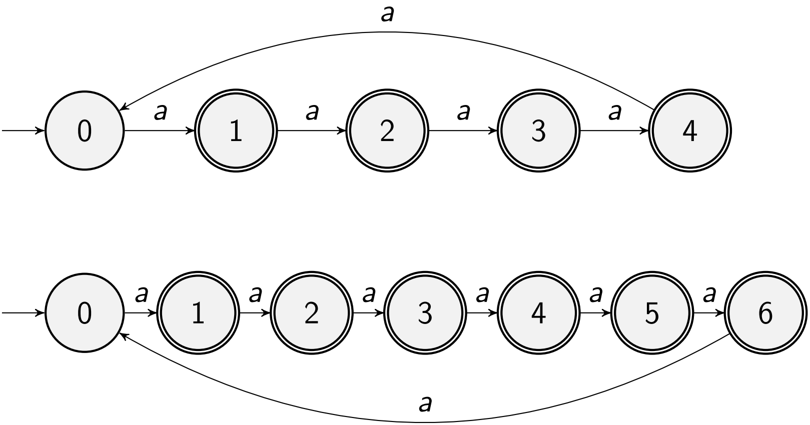
$\exists$ DFA for L_2: 35 states: swap final-$\overline{\text{final}}$ states in DFA for L_1.

Small NFA for $L_2 = \{a^i : i \not\equiv 0 \pmod{35}\}$

Need these two NFA's.



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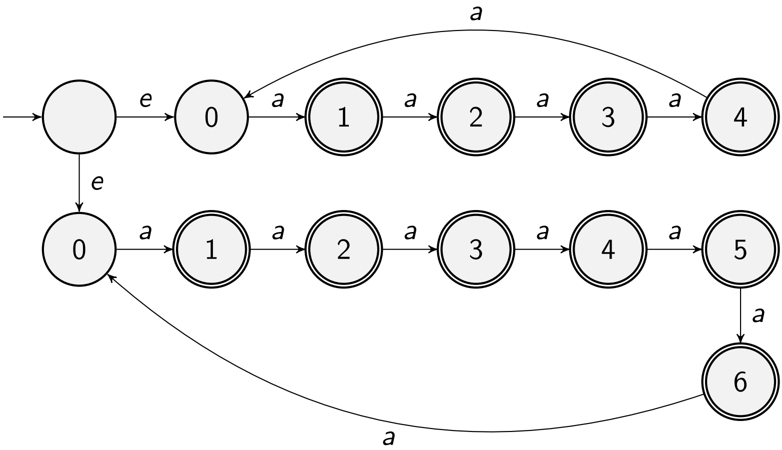

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NFA for L_2 can be done with $1 + 5 + 7 = 13$ states.

Proving That a Language Is Not Regular

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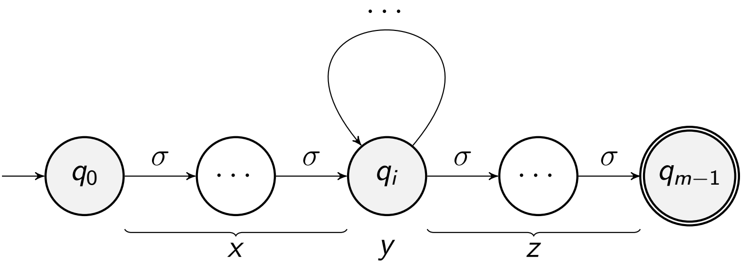

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We then find some i such that $xy^iz \notin L$ for the contradiction.

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Contradiction since $k \geq 1$.

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So what do to?

If L_3 is regular then $L_2 = \overline{L_3}$ is regular. But we know that L_2 is not regular. DONE!

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PL says you can always add some constant k to produce a word in the language.
We omit details.

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3. Decidability of WS1S (we did this).