BILL AND NATHAN START RECORDING

HW 03 Solutions

Problem 1

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See next slide for an exercise YOU SHOULD DO

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Give a DFA for which this does not work.

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Write a DFA for L in table form. Give Q, δ, s, F.
(We already know Σ.)

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Since there are $3 \times 4 \times 5 = 60$ states and $|\Sigma| = 3$, some of you wrote out all 180 transitions in the table.

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You can write the table with 3 transitions using algebra.

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In the future do these problems the easy way.

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We do it in two parts and then combine them.

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For every $n \geq 90$ there is $c, d \in \mathrm{N}$ such that $cx + dy = n$.
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For every $n \geq 101$ there is $c, d \in \mathrm{N}$ such that $cx + dy + 11 = n$.

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We call this NFA M_c (c for chicken).

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This set of DFAs
1) accepts $\{a^i : i \leq 99\}$ and some a^i with $i \geq 101$.
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3) Has $2 + 3 + 5 + 7 = 17$ states.

Finish

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There may be a smaller NFA but not much smaller.