# BILL START RECORDING

<□▶ <□▶ < □▶ < □▶ < □▶ < □▶ < □ > ○ < ○

## **HW 09 Solutions**

A **Leo-Grammar** is a grammar where each rule is of one of the following forms:

(ロト (個) (E) (E) (E) (E) のへの

A **Leo-Grammar** is a grammar where each rule is of one of the following forms:

AB 
ightarrow CDE



A **Leo-Grammar** is a grammar where each rule is of one of the following forms:

- AB 
  ightarrow CDE
- AB 
  ightarrow CD

A **Leo-Grammar** is a grammar where each rule is of one of the following forms:

- AB 
  ightarrow CDE
- AB 
  ightarrow CD
- $A \rightarrow BC$

A **Leo-Grammar** is a grammar where each rule is of one of the following forms:

- AB 
  ightarrow CDE
- AB 
  ightarrow CD
- $A \rightarrow BC$
- $A\to \sigma$

A **Leo-Grammar** is a grammar where each rule is of one of the following forms:

- $AB \rightarrow CDE$
- AB 
  ightarrow CD
- $A \rightarrow BC$
- $A\to \sigma$
- $A \rightarrow e$

A **Leo-Grammar** is a grammar where each rule is of one of the following forms:

▲□▶ ▲□▶ ▲三▶ ▲三▶ 三三 - のへぐ

- AB 
  ightarrow CDE
- $AB \rightarrow CD$
- $A \rightarrow BC$
- $A\to \sigma$
- $A \rightarrow e$

We assume  $\Sigma = \{a, b\}$ .

A **Leo-Grammar** is a grammar where each rule is of one of the following forms:

▲ロ ▶ ▲周 ▶ ▲ ヨ ▶ ▲ ヨ ▶ → 目 → の Q @

 $\begin{array}{l} AB \rightarrow CDE \\ AB \rightarrow CD \\ A \rightarrow BC \\ A \rightarrow \sigma \\ A \rightarrow e \\ \\ \text{We assume } \Sigma = \{a, b\}. \\ L(G) \text{ is the set of strings in } \{a, b\}^* \text{ that } G \text{ generates.} \end{array}$ 

A **Leo-Grammar** is a grammar where each rule is of one of the following forms:

 $\begin{array}{l} AB \rightarrow CDE \\ AB \rightarrow CD \\ A \rightarrow BC \\ A \rightarrow \sigma \\ A \rightarrow e \\ \\ \mbox{We assume } \Sigma = \{a, b\}. \\ L(G) \mbox{ is the set of strings in } \{a, b\}^* \mbox{ that } G \mbox{ generates.} \end{array}$ 

We do this problem without using O-of. We will assume t is large.

<□▶ <□▶ <□▶ < □▶ < □▶ < □▶ = - つへぐ

#### Want f s.t. A Leo-Grammar with t NTs has $\leq f(t)$ rules.

▲□▶▲圖▶▲圖▶▲圖▶ 圖 のへで

Want f s.t. A Leo-Grammar with t NTs has  $\leq f(t)$  rules. AB  $\rightarrow$  CDE



Want f s.t. A Leo-Grammar with t NTs has  $\leq f(t)$  rules. AB  $\rightarrow$  CDE Number of rules of this type:  $t^5$ .

Want f s.t. A Leo-Grammar with t NTs has  $\leq f(t)$  rules.  $AB \rightarrow CDE$  Number of rules of this type:  $t^5$ .  $AB \rightarrow CD$ 

Want f s.t. A Leo-Grammar with t NTs has  $\leq f(t)$  rules.  $AB \rightarrow CDE$  Number of rules of this type:  $t^5$ .  $AB \rightarrow CD$  Number of rules of this type:  $t^4$ .

Want f s.t. A Leo-Grammar with t NTs has  $\leq f(t)$  rules.  $AB \rightarrow CDE$  Number of rules of this type:  $t^5$ .  $AB \rightarrow CD$  Number of rules of this type:  $t^4$ .  $A \rightarrow BC$ 

▲ロ ▶ ▲周 ▶ ▲ ヨ ▶ ▲ ヨ ▶ → 目 → の Q @

Want f s.t. A Leo-Grammar with t NTs has  $\leq f(t)$  rules.  $AB \rightarrow CDE$  Number of rules of this type:  $t^5$ .  $AB \rightarrow CD$  Number of rules of this type:  $t^4$ .  $A \rightarrow BC$  Number of rules of this type:  $t^3$ .

Want f s.t. **A Leo-Grammar with** t **NTs has**  $\leq f(t)$  **rules.**   $AB \rightarrow CDE$  Number of rules of this type:  $t^5$ .  $AB \rightarrow CD$  Number of rules of this type:  $t^4$ .  $A \rightarrow BC$  Number of rules of this type:  $t^3$ .  $A \rightarrow \sigma$ 

Want f s.t. **A Leo-Grammar with** t **NTs has**  $\leq f(t)$  rules.  $AB \rightarrow CDE$  Number of rules of this type:  $t^5$ .  $AB \rightarrow CD$  Number of rules of this type:  $t^4$ .  $A \rightarrow BC$  Number of rules of this type:  $t^3$ .  $A \rightarrow \sigma$  Number of rules of this type: 2t.

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ ○臣 - のへで

Want f s.t. **A Leo-Grammar with** t **NTs has**  $\leq f(t)$  rules.  $AB \rightarrow CDE$  Number of rules of this type:  $t^5$ .  $AB \rightarrow CD$  Number of rules of this type:  $t^4$ .  $A \rightarrow BC$  Number of rules of this type:  $t^3$ .  $A \rightarrow \sigma$  Number of rules of this type: 2t.  $A \rightarrow e$ 

ション ふゆ アメリア メリア しょうくしゃ

Want f s.t. **A Leo-Grammar with** t **NTs has**  $\leq f(t)$  rules.  $AB \rightarrow CDE$  Number of rules of this type:  $t^5$ .  $AB \rightarrow CD$  Number of rules of this type:  $t^4$ .  $A \rightarrow BC$  Number of rules of this type:  $t^3$ .  $A \rightarrow \sigma$  Number of rules of this type: 2t.  $A \rightarrow e$  Number of rules of this type: t.

Want f s.t. A Leo-Grammar with t NTs has  $\leq f(t)$  rules.  $AB \rightarrow CDE$  Number of rules of this type:  $t^5$ .  $AB \rightarrow CD$  Number of rules of this type:  $t^4$ .  $A \rightarrow BC$  Number of rules of this type:  $t^3$ .  $A \rightarrow \sigma$  Number of rules of this type: 2t.  $A \rightarrow e$  Number of rules of this type: t. Number of rules:  $t^5 + t^4 + t^3 + 2t + t < 2t^5$ .

◆□▶ ◆□▶ ◆三▶ ◆三▶ ○○○

Find a function g such that the following is true: The number of Leo-Grammars with t nonterminals is  $\leq g(t)$ .

(ロト (個) (E) (E) (E) (E) のへの

Find a function g such that the following is true: The number of Leo-Grammars with t nonterminals is  $\leq g(t)$ .

A Leo Grammar with t nonterminals is a subset of the set of rules.

◆□▶ ◆□▶ ◆三▶ ◆三▶ ○○ ○○

Find a function g such that the following is true: The number of Leo-Grammars with t nonterminals is  $\leq g(t)$ .

A Leo Grammar with t nonterminals is a subset of the set of rules.

・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・

There are  $\leq 2t^5$  rules.

Find a function g such that the following is true: The number of Leo-Grammars with t nonterminals is  $\leq g(t)$ .

A Leo Grammar with t nonterminals is a subset of the set of rules.

▲□▶ ▲□▶ ▲目▶ ▲目▶ 三日 - のへの

There are  $\leq 2t^5$  rules.

Hence there are  $\leq 2^{2t^5}$  grammars.

Find a function g such that the following is true: The number of Leo-Grammars with t nonterminals is  $\leq g(t)$ .

A Leo Grammar with t nonterminals is a subset of the set of rules.

ション ふゆ アメリア メリア しょうくしゃ

There are  $\leq 2t^5$  rules.

Hence there are  $\leq 2^{2t^5}$  grammars.

We let  $g(t) = 2^{2t^5}$ .

Find a function h such that the following is true:  $\exists w \in \{a, b\}^*$  of length  $\leq h(t)$  such that there is NO Leo-Grammar G with t nonterminals such that  $L(G) = \{w\}$ .

Find a function h such that the following is true:  $\exists w \in \{a, b\}^*$  of length  $\leq h(t)$  such that there is NO Leo-Grammar G with t nonterminals such that  $L(G) = \{w\}$ .

There are  $\leq 2^{2t^5}$  Grammars.

Find a function h such that the following is true:  $\exists w \in \{a, b\}^*$  of length  $\leq h(t)$  such that there is NO Leo-Grammar G with t nonterminals such that  $L(G) = \{w\}$ .

ション ふゆ アメリア メリア しょうくしゃ

There are  $\leq 2^{2t^5}$  Grammars. Let  $h(t) = 3t^5$ .

Find a function h such that the following is true:  $\exists w \in \{a, b\}^*$  of length  $\leq h(t)$  such that there is NO Leo-Grammar G with t nonterminals such that  $L(G) = \{w\}$ .

ション ふゆ アメリア メリア しょうくしゃ

There are  $\leq 2^{2t^5}$  Grammars. Let  $h(t) = 3t^5$ . There are  $2^{3t^5}$  strings of length  $3t^5$ .

Find a function h such that the following is true:  $\exists w \in \{a, b\}^*$  of length  $\leq h(t)$  such that there is NO Leo-Grammar G with t nonterminals such that  $L(G) = \{w\}$ .

There are 
$$\leq 2^{2t^5}$$
 Grammars.  
Let  $h(t) = 3t^5$ .  
There are  $2^{3t^5}$  strings of length  $3t^5$ 

Hence there must be some w of length  $3t^5$  that is not generated by ANY Leo grammar with t NTs.

ション ふゆ アメリア メリア しょうくしゃ

#### **Problem 2a**



$$\Sigma = \{a, b, c, d\}.$$

$$\Sigma = \{a, b, c, d\}.$$
  
Give a CSG for  $\{w : \#_a(w) = \#_b(w) = \#_c(w) = \#_d(w)\}.$ 

 $\Sigma = \{a, b, c, d\}.$ Give a CSG for  $\{w : \#_a(w) = \#_b(w) = \#_c(w) = \#_d(w)\}.$ How many rules does it have?

 $\Sigma = \{a, b, c, d\}.$ Give a CSG for  $\{w : \#_a(w) = \#_b(w) = \#_c(w) = \#_d(w)\}.$ How many rules does it have?

 $S \rightarrow SABCD \mid e$ 



 $\Sigma = \{a, b, c, d\}.$ Give a CSG for  $\{w : \#_a(w) = \#_b(w) = \#_c(w) = \#_d(w)\}.$ How many rules does it have?

 $S \rightarrow SABCD \mid e$  $AB \rightarrow BA$ 

 $\Sigma = \{a, b, c, d\}.$ Give a CSG for  $\{w : \#_a(w) = \#_b(w) = \#_c(w) = \#_d(w)\}.$ How many rules does it have?

 $S \rightarrow SABCD \mid e$  $AB \rightarrow BA \qquad BA \rightarrow AB$ 

 $\Sigma = \{a, b, c, d\}.$ Give a CSG for  $\{w : \#_a(w) = \#_b(w) = \#_c(w) = \#_d(w)\}.$ How many rules does it have?

 $S \rightarrow SABCD \mid e$   $AB \rightarrow BA \qquad BA \rightarrow AB$  $AC \rightarrow CA$ 



 $\Sigma = \{a, b, c, d\}.$ Give a CSG for  $\{w : \#_a(w) = \#_b(w) = \#_c(w) = \#_d(w)\}.$ How many rules does it have?

$S \rightarrow SABCD \mid$	е
AB  ightarrow BA	BA  ightarrow AB
AC  ightarrow CA	$CA \rightarrow AC$

 $\Sigma = \{a, b, c, d\}.$ Give a CSG for  $\{w : \#_a(w) = \#_b(w) = \#_c(w) = \#_d(w)\}.$ How many rules does it have?

$S \rightarrow SABCD \mid e$	
AB  ightarrow BA	BA  ightarrow AB
AC  ightarrow CA	$CA \rightarrow AC$
AD  ightarrow DA	

 $\Sigma = \{a, b, c, d\}.$ Give a CSG for  $\{w : \#_a(w) = \#_b(w) = \#_c(w) = \#_d(w)\}.$ How many rules does it have?

$S  ightarrow SABCD \mid e$		
AB  ightarrow BA	BA  ightarrow AB	
AC  ightarrow CA	$CA \rightarrow AC$	
AD  ightarrow DA	DA  ightarrow AD	

 $\Sigma = \{a, b, c, d\}.$ Give a CSG for  $\{w : \#_a(w) = \#_b(w) = \#_c(w) = \#_d(w)\}.$ How many rules does it have?

$S  ightarrow SABCD \mid e$	ļ
AB  ightarrow BA	BA  ightarrow AB
$AC \rightarrow CA$	$CA \rightarrow AC$
AD  ightarrow DA	DA  ightarrow AD
BC  ightarrow CB	

 $\Sigma = \{a, b, c, d\}.$ Give a CSG for  $\{w : \#_a(w) = \#_b(w) = \#_c(w) = \#_d(w)\}.$ How many rules does it have?

$S \rightarrow SABCD \mid e$	
AB  ightarrow BA	BA  ightarrow AB
AC  ightarrow CA	$CA \rightarrow AC$
AD  ightarrow DA	DA  ightarrow AD
BC  ightarrow CB	$CB \rightarrow BC$

 $\Sigma = \{a, b, c, d\}.$ Give a CSG for  $\{w : \#_a(w) = \#_b(w) = \#_c(w) = \#_d(w)\}.$ How many rules does it have?

$S \rightarrow SABCD ~  e$	
AB  ightarrow BA	BA  ightarrow AB
AC  ightarrow CA	$CA \rightarrow AC$
AD  ightarrow DA	DA  ightarrow AD
BC  ightarrow CB	$CB \rightarrow BC$
BD  ightarrow DB	

 $\Sigma = \{a, b, c, d\}.$ Give a CSG for  $\{w : \#_a(w) = \#_b(w) = \#_c(w) = \#_d(w)\}.$ How many rules does it have?

$S  ightarrow SABCD \mid e$	
AB  ightarrow BA	BA  ightarrow AB
AC  ightarrow CA	$CA \rightarrow AC$
AD  ightarrow DA	DA  ightarrow AD
$BC \rightarrow CB$	$CB \rightarrow BC$
BD  ightarrow DB	DB  ightarrow BD

 $\Sigma = \{a, b, c, d\}.$ Give a CSG for  $\{w : \#_a(w) = \#_b(w) = \#_c(w) = \#_d(w)\}.$ How many rules does it have?

$S  ightarrow SABCD \mid e$	
AB  ightarrow BA	BA  ightarrow AB
AC  ightarrow CA	$CA \rightarrow AC$
AD  ightarrow DA	DA  ightarrow AD
BC  ightarrow CB	CB  ightarrow BC
BD  ightarrow DB	DB  ightarrow BD
CD  ightarrow DC	

 $\Sigma = \{a, b, c, d\}.$ Give a CSG for  $\{w : \#_a(w) = \#_b(w) = \#_c(w) = \#_d(w)\}.$ How many rules does it have?

$S  ightarrow SABCD \mid e$	
AB  ightarrow BA	BA  ightarrow AB
AC  ightarrow CA	$CA \rightarrow AC$
AD  ightarrow DA	DA  ightarrow AD
BC  ightarrow CB	CB  ightarrow BC
BD  ightarrow DB	DB  ightarrow BD
$CD \rightarrow DC$	$DC \rightarrow CD$

 $\Sigma = \{a, b, c, d\}.$ Give a CSG for  $\{w : \#_a(w) = \#_b(w) = \#_c(w) = \#_d(w)\}.$ How many rules does it have?

$S  ightarrow SABCD \mid e$	
AB  ightarrow BA	BA  ightarrow AB
AC  ightarrow CA	$CA \rightarrow AC$
AD  ightarrow DA	DA  ightarrow AD
BC  ightarrow CB	CB  ightarrow BC
BD  ightarrow DB	DB  ightarrow BD
$CD \rightarrow DC$	$DC \rightarrow CD$
A  ightarrow a	

 $\Sigma = \{a, b, c, d\}.$ Give a CSG for  $\{w : \#_a(w) = \#_b(w) = \#_c(w) = \#_d(w)\}.$ How many rules does it have?

\*ロ \* \* @ \* \* ミ \* ミ \* ・ ミ \* の < や

$S  ightarrow SABCD \mid e$	
AB  ightarrow BA	BA  ightarrow AB
AC  ightarrow CA	$CA \rightarrow AC$
AD  ightarrow DA	DA  ightarrow AD
BC  ightarrow CB	$CB \rightarrow BC$
BD  ightarrow DB	DB  ightarrow BD
$CD \rightarrow DC$	$DC \rightarrow CD$
A  ightarrow a	

 $B \rightarrow b$ 

 $\Sigma = \{a, b, c, d\}.$ Give a CSG for  $\{w : \#_a(w) = \#_b(w) = \#_c(w) = \#_d(w)\}.$ How many rules does it have?

\*ロ \* \* @ \* \* ミ \* ミ \* ・ ミ \* の < や

$S  ightarrow SABCD \mid e$	
AB  ightarrow BA	BA  ightarrow AB
AC  ightarrow CA	$CA \rightarrow AC$
AD  ightarrow DA	DA  ightarrow AD
BC  ightarrow CB	$CB \rightarrow BC$
BD  ightarrow DB	DB  ightarrow BD
CD  ightarrow DC	$DC \rightarrow CD$
A  ightarrow a	

 $B \rightarrow b$  $C \rightarrow c$ 

 $\Sigma = \{a, b, c, d\}.$ Give a CSG for  $\{w : \#_a(w) = \#_b(w) = \#_c(w) = \#_d(w)\}.$ How many rules does it have?

▲□▶ ▲□▶ ▲ 臣▶ ▲ 臣▶ ― 臣 … のへぐ

$S  ightarrow SABCD \mid e$	
AB  ightarrow BA	BA  ightarrow AB
AC  ightarrow CA	$CA \rightarrow AC$
AD  ightarrow DA	DA  ightarrow AD
BC  ightarrow CB	CB  ightarrow BC
BD  ightarrow DB	DB  ightarrow BD
$CD \rightarrow DC$	$DC \rightarrow CD$
A  ightarrow a	
B  ightarrow b	
$C \rightarrow c$	

Number of rules: 17.

$$\Sigma = \{a_1, a_2, \ldots, a_n\}.$$

-

$$\Sigma = \{a_1, a_2, \dots, a_n\}.$$
  
Give a CSG for the language
$$\{w \colon \#_{a_1}(w) = \#_{a_2}(w) = \dots = \#_{a_n}(w)\}.$$
$$S \to SA_1A_2A_3 \cdots A_n \mid e$$

▲□▶ ▲□▶ ▲ 臣▶ ▲ 臣▶ ― 臣 … のへぐ

(for future: 2 rules)

$$\begin{split} \Sigma &= \{a_1, a_2, \dots, a_n\}.\\ \text{Give a CSG for the language} \\ &\{w \colon \#_{a_1}(w) = \#_{a_2}(w) = \dots = \#_{a_n}(w)\}.\\ S &\to SA_1A_2A_3 \cdots A_n \mid e\\ \text{(for future: 2 rules)}\\ \text{For all } &\leq i < j \leq n\\ A_iA_j &\to A_jA_i.\\ A_jA_i &\to A_iA_j. \end{split}$$

▲□▶▲□▶▲目▶▲目▶ 目 のへで

$$\begin{split} \Sigma &= \{a_1, a_2, \dots, a_n\}.\\ \text{Give a CSG for the language} \\ &\{w \colon \#_{a_1}(w) = \#_{a_2}(w) = \dots = \#_{a_n}(w)\}.\\ S &\to SA_1A_2A_3 \cdots A_n \mid e\\ \text{(for future: 2 rules)} \\ \text{For all} &\leq i < j \leq n\\ A_iA_j \to A_jA_i.\\ A_jA_i \to A_iA_j.\\ \text{(for future: 2} \binom{n}{2} = n(n-1) \text{ rules}) \end{split}$$

▲□▶ ▲圖▶ ▲≣▶ ▲≣▶ ▲国 ● ● ●

 $\Sigma = \{a_1, a_2, \ldots, a_n\}.$ Give a CSG for the language  $\{w: \#_{a_1}(w) = \#_{a_2}(w) = \cdots = \#_{a_n}(w)\}.$  $S \rightarrow SA_1A_2A_3\cdots A_n \mid e$ (for future: 2 rules) For all < i < j < n $A_i A_i \rightarrow A_i A_i$ .  $A_i A_i \rightarrow A_i A_i$ . (for future:  $2\binom{n}{2} = n(n-1)$  rules) For all 1 < i < n $A_i \rightarrow a_i$ 

 $\Sigma = \{a_1, a_2, \ldots, a_n\}.$ Give a CSG for the language  $\{w: \#_{a_1}(w) = \#_{a_2}(w) = \cdots = \#_{a_n}(w)\}.$  $S \rightarrow SA_1A_2A_3\cdots A_n \mid e$ (for future: 2 rules) For all < i < j < n $A_i A_i \rightarrow A_i A_i$ .  $A_i A_i \rightarrow A_i A_i$ . (for future:  $2\binom{n}{2} = n(n-1)$  rules) For all 1 < i < n $A_i \rightarrow a_i$ (for future: *n* rules.)

・ロト・西・・田・・田・・日・

#### Problem 2c

Find a function r such that your CSG from the last part has  $\leq r(n)$  rules.

(ロト (個) (E) (E) (E) (E) のへの

#### **Problem 2c**

Find a function *r* such that your CSG from the last part has  $\leq r(n)$  rules. 2 +  $n(n-1) + n = 2 + n^2 - n + n = n^2 + 2$ 

▲□▶ ▲□▶ ▲目▶ ▲目▶ 三日 - のへの

#### **Problem 2c**

Find a function *r* such that your CSG from the last part has  $\leq r(n)$  rules.  $2 + n(n-1) + n = 2 + n^2 - n + n = n^2 + 2$ Let  $r(n) = n^2 + 2$ .