BILL START RECORDING

HW 09 Solutions

Problem 1

A **Leo-Grammar** is a grammar where each rule is of one of the following forms:

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We assume $\Sigma = \{a, b\}$.

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$L(G)$ is the set of strings in $\{a, b\}^*$ that G generates.

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We do this problem without using O-of. We will assume t is large.

Problem 1a

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Want f s.t. **A Leo-Grammar with t NTs has $\leq f(t)$ rules.**

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$AB \rightarrow CDE$ Number of rules of this type: t^5.

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$A \rightarrow BC$ Number of rules of this type: t^3.

$A \rightarrow \sigma$ Number of rules of this type: $2t$.

$A \rightarrow e$ Number of rules of this type: t.

Number of rules: $t^5 + t^4 + t^3 + 2t + t \le 2t^5$.

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Hence there are $\leq 2^{2t^5}$ grammars.

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There are $\leq 2t^5$ rules.

Hence there are $\leq 2^{2t^5}$ grammars.

We let $g(t) = 2^{2t^5}$.

Problem 1c

Find a function h such that the following is true:
$\exists \boldsymbol{w} \in \{\boldsymbol{a}, \boldsymbol{b}\}^*$ of length $\leq h(t)$ such that there is NO Leo-Grammar G with t nonterminals such that $L(G) = \{w\}$.

Problem 1c

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Let $h(t) = 3t^5$.

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There are $\leq 2^{2t^5}$ Grammars.
Let $h(t) = 3t^5$.
There are 2^{3t^5} strings of length $3t^5$.

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Find a function h such that the following is true:
$\exists \boldsymbol{w} \in \{\boldsymbol{a}, \boldsymbol{b}\}^*$ of length $\leq h(t)$ such that there is NO Leo-Grammar G with t nonterminals such that $L(G) = \{w\}$.

There are $\leq 2^{2t^5}$ Grammars.
Let $h(t) = 3t^5$.
There are 2^{3t^5} strings of length $3t^5$.

Hence there must be some w of length $3t^5$ that is not generated by ANY Leo grammar with t NTs.

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How many rules does it have?

$S \to SABCD \mid e$

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$S \rightarrow SABCD \mid e$

$AB \rightarrow BA$

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$S \to SABCD \mid e$

$AB \to BA \qquad BA \to AB$

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$AB \rightarrow BA$ $\qquad$ $BA \rightarrow AB$

$AC \rightarrow CA$

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$AC \to CA \qquad CA \to AC$

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$S \to SABCD \mid e$

$AB \to BA$ $\quad$ $BA \to AB$

$AC \to CA$ $\quad$ $CA \to AC$

$AD \to DA$

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$$S \rightarrow SABCD \mid e$$

$$AB \rightarrow BA \qquad BA \rightarrow AB$$

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$$AD \rightarrow DA \qquad DA \rightarrow AD$$

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$AC \to CA$ $\quad$ $CA \to AC$

$AD \to DA$ $\quad$ $DA \to AD$

$BC \to CB$

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$AB \to BA$	$BA \to AB$
$AC \to CA$	$CA \to AC$
$AD \to DA$	$DA \to AD$
$BC \to CB$	$CB \to BC$

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$AB \to BA$ $\quad$ $BA \to AB$

$AC \to CA$ $\quad$ $CA \to AC$

$AD \to DA$ $\quad$ $DA \to AD$

$BC \to CB$ $\quad$ $CB \to BC$

$BD \to DB$

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$S \rightarrow SABCD \mid e$

$AB \rightarrow BA \qquad BA \rightarrow AB$

$AC \rightarrow CA \qquad CA \rightarrow AC$

$AD \rightarrow DA \qquad DA \rightarrow AD$

$BC \rightarrow CB \qquad CB \rightarrow BC$

$BD \rightarrow DB \qquad DB \rightarrow BD$

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$S \to SABCD \mid e$

$AB \to BA$ $\quad$ $BA \to AB$

$AC \to CA$ $\quad$ $CA \to AC$

$AD \to DA$ $\quad$ $DA \to AD$

$BC \to CB$ $\quad$ $CB \to BC$

$BD \to DB$ $\quad$ $DB \to BD$

$CD \to DC$

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$S \to SABCD \mid e$

$AB \to BA$	$BA \to AB$
$AC \to CA$	$CA \to AC$
$AD \to DA$	$DA \to AD$
$BC \to CB$	$CB \to BC$
$BD \to DB$	$DB \to BD$
$CD \to DC$	$DC \to CD$

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$AB \to BA$ $\qquad$ $BA \to AB$

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$BC \to CB$ $\qquad$ $CB \to BC$

$BD \to DB$ $\qquad$ $DB \to BD$

$CD \to DC$ $\qquad$ $DC \to CD$

$A \to a$

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$AB \to BA$ $\quad BA \to AB$

$AC \to CA$ $\quad CA \to AC$

$AD \to DA$ $\quad DA \to AD$

$BC \to CB$ $\quad CB \to BC$

$BD \to DB$ $\quad DB \to BD$

$CD \to DC$ $\quad DC \to CD$

$A \to a$
$B \to b$

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$AB \to BA \qquad BA \to AB$

$AC \to CA \qquad CA \to AC$

$AD \to DA \qquad DA \to AD$

$BC \to CB \qquad CB \to BC$

$BD \to DB \qquad DB \to BD$

$CD \to DC \qquad DC \to CD$

$A \to a$
$B \to b$
$C \to c$

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$BC \to CB$ $\quad$ $CB \to BC$

$BD \to DB$ $\quad$ $DB \to BD$

$CD \to DC$ $\quad$ $DC \to CD$

$A \to a$
$B \to b$
$C \to c$

Number of rules: 17.

Problem 2b

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$$\Sigma = \{a_1, a_2, \ldots, a_n\}.$$

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$\Sigma = \{a_1, a_2, \ldots, a_n\}$.
Give a CSG for the language

$$\{w \colon \#_{a_1}(w) = \#_{a_2}(w) = \cdots = \#_{a_n}(w)\}.$$

$S \to SA_1A_2A_3 \cdots A_n \mid e$
(for future: 2 rules)

Problem 2b

$\Sigma = \{a_1, a_2, \ldots, a_n\}$.
Give a CSG for the language

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$S \to SA_1A_2A_3 \cdots A_n \mid e$
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For all $\leq i < j \leq n$
$A_iA_j \to A_jA_i$.
$A_jA_i \to A_iA_j$.

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$\Sigma = \{a_1, a_2, \ldots, a_n\}$.
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For all $\le i < j \le n$
$A_iA_j \to A_jA_i$.
$A_jA_i \to A_iA_j$.
(for future: $2\binom{n}{2} = n(n-1)$ rules)

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Give a CSG for the language

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For all $\leq i < j \leq n$
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$A_jA_i \to A_iA_j$.
(for future: $2\binom{n}{2} = n(n-1)$ rules)

For all $1 \leq i \leq n$
$A_i \to a_i$

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Give a CSG for the language

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(for future: 2 rules)

For all $\leq i < j \leq n$
$A_iA_j \to A_jA_i$.
$A_jA_i \to A_iA_j$.
(for future: $2\binom{n}{2} = n(n-1)$ rules)

For all $1 \leq i \leq n$
$A_i \to a_i$
(for future: n rules.)

Problem 2c

Find a function r such that your CSG from the last part has $\leq r(n)$ rules.

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$2 + n(n-1) + n = 2 + n^2 - n + n = n^2 + 2$

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Find a function r such that your CSG from the last part has $\leq r(n)$ rules.

$2 + n(n-1) + n = 2 + n^2 - n + n = n^2 + 2$

Let $r(n) = n^2 + 2$.