Regular Expressions

Recognizers vs Generators

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We want to write expressions that **generate** strings.

Regular Expressions over Σ

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We give examples and assign meaning.

Example and Meaning

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Def If α is a regex then $L(\alpha)$ is the set of strings it generates.

Examples

1. $b^*(ab^*ab^*)^*ab^*$
2. $b^*(ab^*ab^*ab^*)^*$
3. $(b^*(ab^*ab^*)^*ab^*) \cup (b^*(ab^*ab^*ab^*)^*)$

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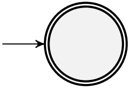

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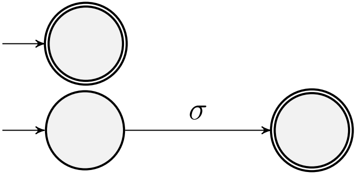

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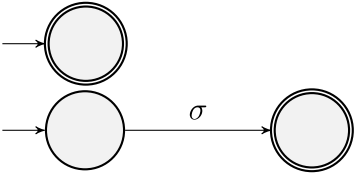

Rest of the proof on next slide.

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Case 3 $\alpha = \alpha_1^*$. Similar. Use closure under Kleene $*$.

How Does Size of NFA and Regex Compare

If α was of length n then the NFA you get for it has $\leq 2n$ states.

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Then the desired regex is

$$E(s, f_1) \cup E(s, f_2) \cup \cdots \cup E(s, f_m)$$

Notation: $\delta(q, w)$

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$$\delta(q, e) = q.$$

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Dynamic Programming We will use all of this information to get our final answer.

Definition of $R(i, j, k)$

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For all $1 \leq i, j \leq n$ $0 \leq k \leq n$, we will find a **regex** for $R(i, j, k)$.

Finding Regex for $R(i,j,k)$

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$$R(i,j,0) = \begin{cases} \{\sigma : \delta(i,\sigma) = j\} & \text{if } i \ne j \ \} \\ \{\sigma : \delta(i,\sigma) = j\} \cup \{e\} & \text{if } i = j \ \} \end{cases} \tag{1}$$

$R(i, j, 0)$ is a Regex. Inductive Step

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We will now **assume** that for all $1 \leq i,j \leq n$, $R(i,j,k-1)$ is a Regex and **prove** that for all $1 \leq i,j \leq n$, $R(i,j,k)$ is a Regex.

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This is both of the following:

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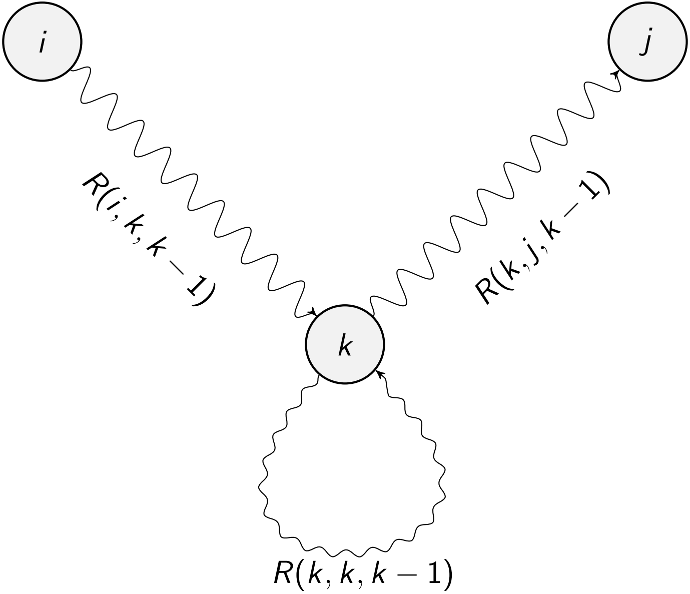
This is both of the following:

1. A proof by induction on k that, for all $1 \le i,j \le n$, $R(i,j,k)$ is a Regex.
2. A dynamic program that computes all $R(i,j,k)$.

Inductive Step $R(i, j, k)$ as a Picture



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If ALL $R(i,j,k-1)$ are Regex, then ALL $R(i,j,k)$ are Regex.

Textbook Regular Expressions

Recall that lang $\{a, b\}^* a \{a, b\}^n$.

1. DFA requires 2^{n+1} states.
2. NFA can be done with $n + 2$ states.
3. How long is the regex for it? Regard the $\{a, b\}^* a$ part to be $O(1)$ length.

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$\{a,b\}^*a\{a,b\}^n$ is a textbook regular expression of length $O(\log n)$.

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1. *e* is a trex. Every $\sigma \in \Sigma$ is a trex.
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3. If α is a trex then α^* is a trex.
4. (This is the new step.) If α is a trex and $n \in \mathbb{N}$ then α^n is a trex. We write n in binary so length is $|\alpha| + \lg n + O(1)$.

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3. If α is a trex then α^* is a trex.
4. (This is the new step.) If α is a trex and $n \in \mathbb{N}$ then α^n is a trex. We write n in binary so length is $|\alpha| + \lg n + O(1)$.

Clearly
there is a regex for L iff there is a trex for L.

Textbook Regular Expressions over Σ

All the cool kids call them **trex**.

Def

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A trex may give a much shorter expression than a regex.

Regex vs Trex For Length

$$L_n = \Sigma^* a \Sigma^n$$

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Need a lower bound for length of regex for L_n.
Can we show that every regex for L_n requires length $f(n)$ for some $f(n)$ where $\log n \ll f(n)$?

Regex vs Trex For Length

Assume there is a regex for L_n of size $f(n)$ (we pick $f(n)$ later).

Regex vs Trex For Length

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Upshot There is a lang L_n with a trex of size $O(\log n)$ but the regex requires $\geq n$. Great! We have a large size difference.

Perl Regex and Java Regex

Regex and trex:

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1. **PRO** Clean mathematical theory, closed under many operations

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   Maybe only people like me care.

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4. Run the DFA M on a text to find where the pattern occurs.