HW Review

November 23, 2019

2. What is the big advantage of Rabin’s Encryption?
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3. What is the big disadvantage of Rabin’s Encryption?
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   **ANS** Breaking Rabin is equivalent to factoring.
3. What is the big disadvantage of Rabin’s Encryption?  
   **ANS** When Alice decodes she may get several possibilities for what the message is.

If Bob is sending **ENGLISH** texts (or something else easily recognized) then when Alice gets several decodings she can tell which one it's supposed to be.

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4. Give a scenario where that disadvantage is not a problem.
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3. What is the big disadvantage of Rabin’s Encryption? **ANS** When Alice decodes she may get several possibilities for what the message is.

4. Give a scenario where that disadvantage is not a problem. **ANS** If Bob is sending ENGLISH texts (or something else easily recognized) then when Alice gets several decodings she can tell which one it’s supposed to be.
We call a set of $N_1, N_2, N_3$ JUSTINIAN if (1) $N_1$ rel prime to $N_2N_3$, (2) $N_2$ rel prime to $N_1N_3$, and (3) $N_3$ rel prime to $N_1N_2$.

Write Pseudocode to do the following: Given $N_1, N_2, N_3$ JUSTINIAN and $x_1, x_2, x_3$, find $x$ such that

\[
\begin{align*}
x &\equiv x_1 \pmod{N_1} \\
x &\equiv x_2 \pmod{N_2} \\
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1. Input $N_1, N_2, N_3, x_1, x_2, x_3$
2. Find the inverse of $N_1N_2 \mod N_3$. We call this $(N_1N_2)^{-1}$. 
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4. Find the inverse of $N_2 N_3 \mod N_1$. We call this $(N_2 N_3)^{-1}$.
5. Output

\[ x_1(N_2 N_3)^{-1}N_2 N_3 + x_2(N_1 N_3)^{-1}N_1 N_3 + x_3(N_1 N_2)^{-1}N_1 N_2. \]
Hw07, Prob 5

(⊕ is + mod 10.) Alice and Bob are doing BG with \( p = 1019 \), 
\( q = 1051 \), \( r = 5432 \), and \( m = 8761 \). What does Bob send?
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**ANS** $N = 1019 \times 1051 = 1070969$. 
(⊕ is + mod 10.) Alice and Bob are doing BG with $p = 1019$, $q = 1051$, $r = 5432$, and $m = 8761$. What does Bob send?

**ANS** $N = 1019 \times 1051 = 1070969$. $(m_1, m_2, m_3, m_4) = (8, 7, 6, 1)$. 

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$r = 5432$. 

$x_1 = 5432^2 \equiv 590461 \pmod{N}$, hence $b_1 = 1$.

$x_2 = 590461^2 \equiv 944261 \pmod{N}$, hence $b_2 = 1$.

$x_3 = 944261^2 \equiv 20985 \pmod{N}$, hence $b_3 = 5$.

$x_4 = 20985^2 \equiv 201966 \pmod{N}$, hence $b_4 = 6$.

Bob computes the following (all arithmetic mod 10)

$$((b_1 + m_1, b_2 + m_2, b_3 + m_3, b_4 + m_4) \equiv (9, 8, 1, 7) \pmod{10}).$$

He then sends $((9, 8, 1, 7), 268853)$. 


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- \( r = 5432 \).
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- \( x_2 = 590461^2 \equiv 944261 \) (mod \( N \)), hence \( b_2 = 1 \).
- \( x_3 = 944261^2 \equiv 20985 \) (mod \( N \)), hence \( b_3 = 5 \).

Bob computes the following (all arithmetic mod 10)

\[(b_1 + m_1, b_2 + m_2, b_3 + m_3, b_4 + m_4) = (1 + 8, 1 + 7, 5 + 6, 6 + 1) \equiv (9, 8, 1, 7) \pmod{10}.

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Recall the following key exchange protocol:

1. Alice generates rand prime $p$ of length $L$, rand $S \times S$ matrix $A$ over $\mathbb{Z}_p$. You can assume $A$ is invertible.
2. Alice sends $(p, A, HAHA)$. All public.
3. Alice generates rand row $\vec{y} \in \mathbb{Z}_p^S$. Sends $\vec{y}A$.
4. Bob generate rand column $\vec{x} \in \mathbb{Z}_p^S$, Sends $A\vec{x}$.
5. Alice computes $\vec{y}(A\vec{x}) = \vec{y}A\vec{x}$.
6. Bob computes $(\vec{y}A)\vec{x} = \vec{y}A\vec{x}$.
7. Alice and Bob have shared secret $\vec{y}A\vec{x}$.

Eve sees $(p, A, HAHA, \vec{y}A, A\vec{x})$. Help Eve recover $\vec{y}A\vec{x}$.

**ANS**

Eve $A^{-1}$, and then $A^{-1}A\vec{x} = \vec{x}$. Eve knows $\vec{x}$ and $\vec{y}A$ so she can compute $\vec{y}A\vec{x}$. 
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Eve uses $A^{-1}$, and then $A^{-1}A\vec{x} = \vec{x}$. Eve knows $\vec{x}$ and $\vec{y}A$ so she can compute $\vec{y}A\vec{x}$. 
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**ANS** Eve $A^{-1}$, and then $A^{-1}A\vec{x} = \vec{x}$. Eve knows $x$ and $\vec{y}A$ so she can compute $\vec{y}A\vec{x}$. 
A and B do DH over $\mathbb{N}$. 

We look at DIFF problems than the HW looked at.
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2. Alice picks a random \( g \in \{2, \ldots, S\} \) and broadcasts \( g \).

3. Alice picks a random \( a \in \{2, \ldots, T\} \) and broadcasts \( g^a \).

4. Bob picks a random \( b \in \{2, \ldots, T\} \) and broadcasts \( g^b \).

5. Alice computes \( (g^b)^a = g^{ab} \).

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7. The shared secret key is \( g^{ab} \).

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What are the PROS and CONS of doing DH over $\mathbb{N}$?
A and B do DH over $\mathbb{N}$.

1. Security parameters are $S$, $T$.
2. Alice picks a random $g \in \{2, \ldots, S\}$ and broadcasts $g$.
3. Alice picks a random $a \in \{2, \ldots, T\}$ and broadcasts $g^a$.
4. Bob picks a random $b \in \{2, \ldots, T\}$ and broadcasts $g^b$.
5. Alice computes $(g^b)^a = g^{ab}$.
6. Bob computes $(g^a)^b = g^{ab}$.
7. The shared secret key is $g^{ab}$.

We Look at DIFF Problems than the HW looked at
What are the PROS and CONS of doing DH over $\mathbb{N}$?
ANSWERS

Can still use repeated squaring. So if count each operation as one step, still fast.

CAVEAT: Numbers could get big, making the one-op = one step no longer true.

CON: With big numbers, might take too much time or space.

BIGGEST CON: Contrast: Discrete Log seems hard over $\mathbb{Z}_p$ since $x \rightarrow g^x$ is random-looking. But Log over $\mathbb{N}$ is very easy since is monotone. Binary Search works very well. And there are faster ways (e.g., Taylor series)
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Want to factor 81072007 with QS
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Want to factor 81072007 with QS

- Note that $\lceil \sqrt{81072007} \rceil = 9004$.
- Will factor $(9004 + x)^2$ instead of $B$-factor.

The problem had three parts which culminate in obtaining a factor. We do each part on a different slide.
Want to factor 81072007 with QS

- Note that $\lceil \sqrt{81072007} \rceil = 9004$.
- Will factor $(9004 + x)^2$ instead of $B$-factor.
- We only use $x$ such that $(9004 + x)^2 - 81072007 \equiv 0 \pmod{6}$. This way we know that 2 and 3 both divide $(9004 + x)^2$ so it has some factors.
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The problem had three parts which culminate in obtaining a factor. We do each part on a diff slide.
Find which \( x \) to Use

Find \( X \subseteq \{0, 1, 2, 3, 4, 5\} \) such that:

\[
((9004 + x)^2 - 81072007 \equiv 0 \pmod{6}) \iff (x \mod 6 \in X).
\]
Find which $x$ to Use

Find $X \subseteq \{0, 1, 2, 3, 4, 5\}$ such that:

$$((9004 + x)^2 - 81072007 \equiv 0 \pmod{6}) \text{IFF} (x \mod 6 \in X).$$

**ANS**

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**ANS**

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Mod down:
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Find $X \subseteq \{0, 1, 2, 3, 4, 5\}$ such that:

\[
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\]

**ANS**

\[(9004 + x)^2 - 81072007 \equiv 0 \pmod{6}\]

Mod down:

\[(4 + x)^2 - 1 \equiv 0 \pmod{6}\]
Find which \( x \) to Use

Find \( X \subseteq \{0, 1, 2, 3, 4, 5\} \) such that:

\[
((9004 + x)^2 - 81072007 \equiv 0 \pmod{6})) \text{IFF} (x \mod 6 \in X).
\]

**ANS**

\[
(9004 + x)^2 - 81072007 \equiv 0 \pmod{6}
\]

Mod down:

\[
(4 + x)^2 - 1 \equiv 0 \pmod{6}
\]

Need to know when \( (x + 4)^2 \equiv 1 \pmod{6} \). Try 0, 1, 2, 3, 4, 5 and find \( X = \{1, 3\} \).
Factor \((9004 + x)^2 - 81072007\)

For \(x \geq 0\) with \(x \mod 6 \in X\), factor \((9004 + x)^2 - 81072007\) until have what is needed for QS.
Factor $(9004 + x)^2 - 81072007$

For $x \geq 0$ with $x \mod 6 \in X$, factor $(9004 + x)^2 - 81072007$ until have what is needed for QS.

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AH-HA! - we can mult first and fourth row

\[(9005 \times 9013)^2 \equiv 2^2 \times 3^6 \times 7^2 \times 11^2 \times 13^2\]
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Use Last Part to Factor

$$(9005 \times 9013)^2 \equiv (2 \times 3^3 \times 7 \times 11 \times 13)^2$$
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\[81162065^2 \equiv 54054^2 \pmod{81072007}\]

GCD \((36004, 81072007) = 9001\) is a factor. Done!
Use Last Part to Factor

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\[81162065^2 \equiv 54054^2 \pmod{81072007}\]

Mod down:

\[90058^2 \equiv 54054^2\]
Use Last Part to Factor

\[(9005 \times 9013)^2 \equiv (2 \times 3^3 \times 7 \times 11 \times 13)^2\]

\[81162065^2 \equiv 54054^2 \pmod{81072007}\]

Mod down:

\[9005^2 \equiv 54054^2\]

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Use Last Part to Factor

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\[GCD(36004, 81072007) = 9001\] is a factor. Done!
Hw09, Prob 2: Commentary

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and we said AH HA: The First and Fourth Column.

The first and fourth both have 2,3,7,11,13 as primes. Having same primes NOT always the case. The following could happen:

\[
\begin{array}{c}
1 & 2^2 \times 3 \times 7 \times 11 \\
2 & 3 \times 7^2 \times 11 \\
3 & 7^3 \times 11^8 \times 13^2
\end{array}
\]

Mult all three to get

\[
2^2 \times 3^2 \times 7^6 \times 11^{10} \times 13^2 = (2 \times 3 \times 7^3 \times 11^5)^2
\]
Eve wants to factor $G = 139, 323, 391$ (a product of two primes) using the method Golomb used to factor the Jevons number. This problem will guide you through it.
Hw09, Prob 3: Setup

Eve wants to factor \( G = 139, 323, 391 \) (a product of two primes) using the method Golomb used to factor the Jevons number. This problem will guide you through it.

Throughout this problem \( x, y \) are such that \( G = x^2 - y^2 \). During this problem we reduce the number of options for \((x, y)\).
Eve wants to factor $G = 139, 323, 391$ (a product of two primes) using the method Golomb used to factor the Jevons number. This problem will guide you through it.

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Each part is on its own slide.
Find $d$

Find a $0 \leq d \leq 99$ such that the following holds:

$$y^2 \equiv x^2 + d \pmod{100}$$
Find $d$

Find a $0 \leq d \leq 99$ such that the following holds:

$$y^2 \equiv x^2 + d \pmod{100}$$

ANS

$$G = x^2 - y^2$$
Find \( d \)

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\[
y^2 \equiv x^2 + d \pmod{100}
\]

ANS

\[
G = x^2 - y^2
\]

\[
91 \equiv x^2 - y^2 \pmod{100}
\]
Find \( d \)

Find a \( 0 \leq d \leq 99 \) such that the following holds:

\[
y^2 \equiv x^2 + d \pmod{100}
\]

\textbf{ANS}

\[
G = x^2 - y^2
\]

\[
91 \equiv x^2 - y^2 \pmod{100}
\]

\[
y^2 \equiv x^2 + 9 \pmod{100}
\]
Find $d$

Find a $0 \leq d \leq 99$ such that the following holds:

$$y^2 \equiv x^2 + d \pmod{100}$$

**ANS**

$$G = x^2 - y^2$$

$$91 \equiv x^2 - y^2 \pmod{100}$$

$$y^2 \equiv x^2 + 9 \pmod{100}$$

$$d = 9.$$
List all possibilities for \((x^2 \mod 100, y^2 \mod 100)\).
Possible \((x, y)\)

List all possibilities for \((x^2 \mod 100, y^2 \mod 100)\).

**ANS**
The following is the set of all squares \(\mod 100\)

\[
\{0, 1, 4, 9, 16, 21, 24, 25, 29, 36, 41, 44, 49\} \cup \\
\{56, 61, 64, 69, 76, 81, 84, 89, 96\}
\]
Possible \((x, y)\)

List all possibilities for \((x^2 \text{ mod } 100, y^2 \text{ mod } 100)\).

**ANS**

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Need all pairs that differ by 9 mod 100:
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\{0, 1, 4, 9, 16, 21, 24, 25, 29, 36, 41, 44, 49\} \cup \\
\{56, 61, 64, 69, 76, 81, 84, 89, 96\}
\]

Need all pairs that differ by 9 mod 100:

\[
\{(0, 9), (16, 25)\}
\]
Narrow Down Search for $x$

Find all $x \pmod{100}$ such that $x^2 \equiv 0 \pmod{100}$ OR $x^2 \equiv 16 \pmod{100}$. Put the union of the two sets into numeric order. Note that at the end you will have a small set $A$ such that

$$x \pmod{100} \in A.$$
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**ANS**

$x^2 \equiv 0 \pmod{100}$ has solutions

$$\{0, 10, 20, 30, 40, 50, 60, 70, 80, 90\}$$
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Putting it together we get

$$\{0, 4, 10, 20, 30, 40, 46, 50, 54, 60, 70, 80, 90, 96\}$$
Bound \( x \)

Show that \( x \geq \sqrt{G} \)

**ANS**

\( G = x^2 - y^2 \) so
Show that $x \geq \sqrt{G}$

**ANS**

$G = x^2 - y^2$ so

$$x^2 = G + y^2$$

pause

$$x = \sqrt{G + y^2} \geq \sqrt{G}.$$
Factor $G$

Complete the algorithm and factor $G$.

ANS

$x \mod 100 \in \{0, 4, 10, 20, 30, 40, 46, 50, 54, 60, 70, 80, 90, 96\}$
**Factor** $G$

Complete the algorithm and factor $G$.

**ANS**

$x \mod 100 \in \{0, 4, 10, 20, 30, 40, 46, 50, 54, 60, 70, 80, 90, 96\}$

$x \geq \sqrt{G} = 11803$
**Factor $G$**

Complete the algorithm and factor $G$.

**ANS**

$x \mod 100 \in \{0, 4, 10, 20, 30, 40, 46, 50, 54, 60, 70, 80, 90, 96\}$

$x \geq \sqrt{G} = 11803$

$y = \sqrt{x^2 - 139323391}$
Complete the algorithm and factor \( G \).

**ANS**

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Table that tries out all \( x \geq 11803 \)

\[ 11909 \times 11699 = 139323391 \]
Factor $G$

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WOW- that table ended very fast! $x = 11804$, $y = 105$
Factor $G$

Complete the algorithm and factor $G$.

**ANS**

$x \mod 100 \in \{0, 4, 10, 20, 30, 40, 46, 50, 54, 60, 70, 80, 90, 96\}$

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WOW- that table ended very fast! $x = 11804, y = 105$

$x^2 - y^2 = 139323391$
Factor $G$

Complete the algorithm and factor $G$.

ANS

$x \mod 100 \in \{0, 4, 10, 20, 30, 40, 46, 50, 54, 60, 70, 80, 90, 96\}$

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WOW- that table ended very fast! $x = 11804$, $y = 105$

$x^2 - y^2 = 139323391$

$(x + y)(x - y) = 139323391$
Complete the algorithm and factor \( G \).

**ANS**

\[
x \mod 100 \in \{0, 4, 10, 20, 30, 40, 46, 50, 54, 60, 70, 80, 90, 96\}
\]

\[
x \geq \sqrt{G} = 11803
\]

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y = \sqrt{x^2 - 139323391}
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\[
x^2 - y^2 = 139323391
\]

\[
(x + y)(x - y) = 139323391
\]

\[
(11804 + 105)(11804 - 105) = 139323391
\]
Factor $G$

Complete the algorithm and factor $G$.

**ANS**

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WOW- that table ended very fast! $x = 11804$, $y = 105$

$x^2 - y^2 = 139323391$

$(x + y)(x - y) = 139323391$

$(11804 + 105)(11804 - 105) = 139323391$

$11909 \times 11699 = 139323391$
Similar to Quad Sieve?

Want to factor $G$. 

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Golumb’s algorithm looks for $x, y$ such that $x^2 - y^2 = G$. The key is to cut down on the pairs $(x, y)$ to check.
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Golumb’s algorithm looks for $x, y$ such that $x^2 - y^2 = G$. The key is to cut down on the pairs $(x, y)$ to check.

Quad Sieve Alg looks for $x, y$ such that $x^2 - y^2 \equiv 0 \pmod{G}$. They key is to use Quad Sieve Method ($B$-factoring) to create $x, y$. 
Hw10, Part 1

READ MIDTERM SOLUTIONS
We want to factor 91 by QS by hand.

The problem did this with $B = 1$ and $B = 2$.

We do this on the next two slides.
Hw10, Prob2a, Factoring 91 with $B = 1$

ANS

$(10 + 0)^2 = 100 \equiv 9$ NOT 1-fact.

$(10 + 1)^2 = 121 \equiv 30 = 2 \times 15$ NOT 1-fact.

$(10 + 2)^2 = 144 \equiv 53$ NOT 1-fact.

$(10 + 3)^2 = 169 \equiv 78 = 2 \times 39$ NOT 1-fact.

$(10 + 4)^2 = 196 \equiv 14 = 2 \times 7$ NOT 1-fact.

$(10 + 5)^2 = 225 \equiv 43$ NOT 1-fact.

$(10 + 6)^2 = 256 \equiv 74 = 2 \times 37$ NOT 1-fact.

$(10 + 7)^2 = 289 \equiv 16 = 2^4 = 4^2$ YES! 1-fact.

List length 8. And great: it’s a square!
Hw10, Prob2a, Factoring 91 with $B = 1$

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$(10 + 6)^2 = 256 \equiv 74 = 2 \times 37$ NOT 1-fact.
$(10 + 7)^2 = 289 \equiv 16 = 2^4 = 4^2$ YES! 1-fact.
List length 8. And great: it’s a square!
$17^2 - 4^2 \equiv 0$ (mod 91)
Hw10, Prob2a, Factoring 91 with $B = 1$

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List length 8. And great: it’s a square!

$17^2 - 4^2 \equiv 0 \pmod{91}$

$(17 - 4)(17 + 4) \equiv 0 \pmod{91}$
ANS

\[(10 + 0)^2 = 100 \equiv 9 \text{ NOT 1-fact.}\]
\[(10 + 1)^2 = 121 \equiv 30 = 2 \times 15 \text{ NOT 1-fact.}\]
\[(10 + 2)^2 = 144 \equiv 53 \text{ NOT 1-fact.}\]
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\[(10 + 7)^2 = 289 \equiv 16 = 2^4 = 4^2 \text{ YES! 1-fact.}\]

List length 8. And great: it’s a square!

\[17^2 - 4^2 \equiv 0 \pmod{91}\]
\[(17 - 4)(17 + 4) \equiv 0 \pmod{91}\]
\[13 \times 21 \equiv 0 \pmod{91}\]
Hw10, Prob2a, Factoring 91 with $B = 1$

ANS

$(10 + 0)^2 = 100 \equiv 9 \not\equiv 1$ NOT 1-fact.
$(10 + 1)^2 = 121 \equiv 30 = 2 \times 15$ NOT 1-fact.
$(10 + 2)^2 = 144 \equiv 53$ NOT 1-fact.
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$17^2 - 4^2 \equiv 0 \pmod{91}$
$(17 - 4)(17 + 4) \equiv 0 \pmod{91}$
$13 \times 21 \equiv 0 \pmod{91}$

$GCD(13, 91) = 13$, So 13 is a factor.
Hw10, Prob2a, Factoring 91 with $B = 1$

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$(10 + 0)^2 = 100 \equiv 9 \text{ NOT } 1\text{-fact.}$
$(10 + 1)^2 = 121 \equiv 30 = 2 \times 15 \text{ NOT } 1\text{-fact.}$
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List length 8. And great: it’s a square!

$17^2 - 4^2 \equiv 0 \pmod{91}$
$(17 - 4)(17 + 4) \equiv 0 \pmod{91}$
$13 \times 21 \equiv 0 \pmod{91}$

$GCD(13, 91) = 13$, So 13 is a factor.

Could have also done $GCD(21, 91) = 7$ is a factor.
ANS
$(10 + 0)^2 ≡ 100 ≡ 9 = 3^2 \pmod{91}$. List size 1. And is a square!

$$100 - 9 ≡ 0 \pmod{91}$$

$$10^2 - 3^2 ≡ 0 \pmod{91}$$

$$(10 - 3)(10 + 3) ≡ 0 \pmod{91}$$

$$7 \times 13 ≡ 0 \pmod{91}$$

$GCD(7, 91) = 7$, so 7 is a factor.
Could also have done $GCD(13, 91) = 13$ is a factor.
ANS

$B = 1$ QS took 8 steps
$B = 2$ QS took 1 step

Trivial takes a step for 2, 3, 4, 5, 6, 7, so 6 steps.
Trivial is better than $B = 1$ but not as good as $B = 2$. 
Hw10, Prob 3

We want to factor $N = 53044897$ via QS. We will use $B = 14$. 
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We are going to use the log-trick to detect ahead of time if a number is probably \( B \)-fact. We will use \( \log_{10} \).

Recall the trick: For each \( y \) that we are pondering \( B \)-factoring, compute

\[
\left\lceil \log_{10}(y) \right\rceil - \sum_{p \text{ div } y} \left\lceil \log_{10}(p) \right\rceil.
\]

(the div \( y \) goes through the first \( B \) primes)
If big then \( y \) is \( B \)-factorable, if small then not.
Hw10, Prob 3

We want to factor $N = 53044897$ via QS. We will use $B = 14$.

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We try to determine what big and small should be.
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(the div $y$ goes through the first $B$ primes)
If big then $y$ is $B$-factorable, if small then not.

We try to determine what big and small should be.

Need to know: $\left\lceil \sqrt{N} \right\rceil = 7284.$
First 14 primes: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43.
Take the first 20 numbers $y$ of QS and compute

$$[\log_{10}(y)] - \sum_{p \text{ div } y} [\log_{10}(p)].$$

For each one, see if it is $B$-factorable. Determine what big and small should be.
Hw10, Prob3, The Problem

Take the first 20 numbers $y$ of QS and compute

$$\left\lceil \log_{10}(y) \right\rceil - \sum_{p \text{ div } y} \left\lceil \log_{10}(p) \right\rceil.$$

For each one see if it IS $B$-factorable.
Take the first 20 numbers $y$ of QS and compute

$$\left\lfloor \log_{10}(y) \right\rfloor - \sum_{p \text{ div } y} \left\lfloor \log_{10}(p) \right\rfloor .$$

For each one see if it IS $B$-factorable.

Determine what big and small should be.
1. \((7284 + 1)^2 \equiv 26328\). primes: 2, 3. Diff: \(5 - 1 - 1 = 3\).
\((7284 + 1)^2 \equiv 26328 = 2^3 \times 3 \times 1097\). NOT \(B\)-fact.

2. \((7284 + 4)^2 \equiv 70047\). primes: 3, 43. Diff: \(5 - 1 - 2 = 2\).
\((7284 + 4)^2 \equiv 70047 = 3^2 \times 43 \times 181\). NOT \(B\)-fact.

3. \((7284 + 5)^2 \equiv 84624\). primes: 2, 3, 41, 43. Diff:
\(5 - 1 - 1 - 2 - 2 = -1\).
\((7284 + 5)^2 \equiv 84624 = 2^4 \times 3 \times 41 \times 43\). IS \(B\)-fact.

4. \((7284 + 7)^2 \equiv 113784\). primes: 2, 3, 11. Diff: \(6 - 1 - 1 - 2 = 2\).
\((7284 + 7)^2 \equiv 113784 = 2^3 \times 3 \times 11 \times 431\). NOT \(B\)-fact.

5. \((7284 + 8)^2 \equiv 128367\). primes: 3, 17. Diff: \(6 - 1 - 2 = 3\).
\((7284 + 8)^2 \equiv 128367 = 3^2 \times 17 \times 839\). NOT \(B\)-fact

6. \((7284 + 10)^2 \equiv 157539\). primes: 3, 17. Diff: \(6 - 1 - 2 = 3\).
\((7284 + 10)^2 \equiv 157539 = 3 \times 17 \times 3089\). NOT \(B\)-fact
1. \((7284 + 11)^2 \equiv 172128\). primes: 2,3,11. Diff:
\[6 - 1 - 1 - 2 = 2.\]
\((7284 + 11)^2 \equiv 172128 = 2^5 \times 3 \times 11 \times 163\). NOT B-fact

2. \((7284 + 13)^2 \equiv 201312\). primes: 2,3. Diff: \(6 - 1 - 1 = 4\).
\((7284 + 13)^2 \equiv 201312 = 2^5 \times 3^3 \times 233\). NOT B-fact

3. \((7284 + 17)^2 \equiv 259704\). primes: 2,3. Diff: \(6 - 1 - 1 = 4\).
\((7284 + 17)^2 \equiv 259704 = 2^3 \times 3^2 \times 3607\). NOT B-fact

4. \((7284 + 19)^2 \equiv 288912\). primes: 2,3,13. Diff:
\[6 - 1 - 1 - 2 = 2.\]
\((7284 + 19)^2 \equiv 288912 = 2^4 \times 3 \times 13 \times 463\). NOT B-fact

We can take \(s = -1\). \(s = 0\) and \(s = 1\) also work and are probably better.
Hw11, Prob 1

READ MIDTERM SOLUTIONS
Zelda wants to do (7, 14) information-theoretic secret sharing. The players are \( A_1, \ldots, A_{14} \). The secret string is 1001.
Zelda wants to use the random string method. How many strings does $A_1$ get? How long are the strings $A_1$ gets?

**ANS**

$A_1$ will get a string for EVERY 7-sized set she is a member of. So that will be

\[
\binom{13}{6} = \frac{13!}{6!7!} = \frac{13 \times 12 \times 11 \times 10 \times 9 \times 8}{6 \times 5 \times 4 \times 3 \times 2}
\]

The 12 cancels with the $3 \times 4$. The 10 cancels the $2 \times 5$. So we end up with:

\[
\frac{13 \times 11 \times 9 \times 8}{6} = 13 \times 11 \times 3 \times 4 = 1716.
\]

So there are 1716 strings. Each string is the same length as the secret, so that is length 4.
Zelda wants to use the polynomial method. What is the smallest prime Zelda can use? What is the degree of the polynomial that Zelda uses? How many strings does $A_1$ get? How long are they?

**ANS**

We need a prime $p$ such that $2^4 < p$, so we take 17. The degree is 6 since 7 points determine a 6th degree polynomial. $A_1$ gets just one string. The string is in $\mathbb{Z}_{17}$ padded out to length 4.
Hw11, Prob 3: The Problem

DESCRIBE the random-string (3, 9) secret sharing scheme. You must describe both what Zelda gives out, and how any three people can determine the secret.

How long are the string $A_1$ gets? How many strings does $A_1$ get?

DO AN EXAMPLE (we omit this in the solution).
ANS
We call the people $A_1, \ldots, A_9$. We give the $(3, 9)$ secret sharing method via random strings.

1. Zelda has secret $s$.

2. For every $1 \leq i < j < k \leq 9$, Zelda generates two random strings: $r_{i, j, k, i}$, $r_{i, j, k, j}$. We now visit every triple and say what Zelda gives them. All of the players will be visited many times. How many? $\binom{8}{2}$ which is the number of triples they are in.
Let $1 \leq i < j < k \leq 9$.
   ▶ Give $A_i$ the string $(i, j, k, r_{i, j, k, i})$
   ▶ Give $A_j$ the string $(i, j, k, r_{i, j, k, j})$
   ▶ Give $A_k$ the string $(i, j, k, r_{i, j, k, i} \oplus r_{i, j, k, j} \oplus s)$

3. If $A_i, A_j, A_k$ get together they will compute

$$r_{i, j, k, i} \oplus r_{i, j, k, j} \oplus (r_{i, j, k, i} \oplus r_{i, j, k, j} \oplus s) = s$$
$A_1$ gets strings of length $n$, the same size as the secret.

$A_1$ gets $\binom{8}{2} = 28$ strings.
DESCRIBE the polynomial (4, 7) secret sharing scheme. You must describe both what Zelda gives out, and how any four people can determine the secret.

How many strings does each person get?

DO AN EXAMPLE of your method. (We omit this in the solution.)
1. Zelda has secret $s$.

2. Zelda finds a prime $p$ such that $p > 2|s|$.
   Zelda generates random $r_3, r_2, r_1 \in \{0, \ldots, p - 1\}$.
   Zelda forms polynomial

   $$p(x) = r_3x^3 + r_2x^2 + r_1x + s$$

   ($s$ was a string of 0’s and 1’s. We now view it as a number written in binary)

3. For all $1 \leq i \leq 7$, give $A_i$ the number $p(i) \pmod{p}$.

4. If any four get together they have four points on a cubic. Hence they can recover the entire cubic, and hence the constant term which is the secret.
Everyone gets a string of length $n$, the length of the secret.

Everyone gets just one such string.
READ MIDTERM SOLUTIONS
Zelda does (3,5) secret sharing. The secret is of length 2, so they use the prime 5. Zelda gives out the following numbers:

A_1 gets 3,
A_2 gets 3,
A_3 gets 3,
A_4 gets 3,
A_5 gets 3.

(You prob think the secret is 3. You are correct.)
Hw12, Prob 2a: The Problem

1. $A_1$ and $A_2$ get together. Show that for $c = 0, 1, 2$, there is a quadratic polynomial over $\mathbb{Z}_5$ where ALL of the following hold:

   1.1 $f(1) = 3$
   1.2 $f(2) = 3$
   1.3 The constant term is $c$ (which is equivalent to $f(0) = c$).

   (NOTE: it’s also true for $c = 3, 4$ but I want to spare you the work. This is important because, if you did the problem with $c = 0, 1, 2, 3, 4$ you would show that $A_1$ and $A_2$ have learned NOTHING since all secrets are still possible.) Show your work.
ALL ≡ are mod 5.
c = 0: \( f(x) \equiv r_2x^2 + r_1x + 0 \). Need \( r_1, r_2 \) so that \( f(1) \equiv 3 \) and \( f(2) \equiv 3 \).
f(1) = 3: \( r_2 + r_1 + 0 \equiv 3 \), so \( r_2 + r_1 \equiv 3 \).
f(2) = 3: \( 4r_2 + 2r_1 + 0 \equiv 3 \), so \( 4r_2 + 2r_1 \equiv 3 \).
Algebra shows \( r_2 = 1 \) and \( r_1 = 2 \), so \( f(x) \equiv x^2 + 2x \).

c = 1: Omitted, similar.
c = 2: Omitted, similar.
Problem: What is the secret.

Answer: ALL \equiv \text{ are mod 5.}

\( f(x) \equiv r_2 x^2 + r_1 x + s \)

\( f(1) \equiv 3 \) so \( r_2 + r_1 + s \equiv 3 \)

\( f(2) \equiv 3 \) so \( 4r_2 + 2r_1 + s \equiv 3 \)

\( f(3) \equiv 3 \) so \( 9r_2 + 3r_1 + s \equiv 3 \), so \( 4r_2 + 3r_1 + s \equiv 3 \).

From these can deduce \( s = 3 \).
Show that there is NO way to do \((t, m)\) Verifiable Secret Sharing in a way that is information-theoretic secure.

*WARNING:* The scheme I showed in class for VSS was comp-secure. This has NO bearing on our problem. Just because there IS a comp-secure scheme does not mean that there is not an info-secure scheme. DO NOT MAKE THIS MISTAKE!!!!!!!
Hw12, Prob 3: The Answer

Assume that there is a \((t, m)\)-VSS scheme. We show that if the players have unlimited computational power then \(t - 1\) can crack the secret. (In fact, 1 can crack the secret but we leave that for you to figure out.)

\(A_1, \ldots, A_{t-1}\) get together. They reveal their shares \(s_1, \ldots, s_{t-1}\). They can find the share of \(A_t\) as follows:

They know the share is of string in \(\{0, 1\}^*\). Let \(\{0, 1\}^*\) be, in lex order, \(u_1, u_2, u_3, \ldots\).

\(A_1, \ldots, A_{t-1}\) try to verify that \(A_t\)'s share is \(u_1\). If they fail they try to verify \(u_2\). Etc. Eventually they will find the share and verify it. They then have \(A_t\)'s share so can crack the secret.