BILL, RECORD LECTURE!!!!

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Go Over Problems 4 and 6 from HW 01

October 10, 2020

The One-Time Pad and Trying to Fake It—and Failing to

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- Correctness:

$$\begin{aligned} Dec_k(Enc_k(m)) &= k \oplus (k \oplus m) \\ &= (k \oplus k) \oplus m \\ &= m \end{aligned}$$

Example Of One-Time Pad

Key is 100010100010001111101111100

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Key is 10001010001000111110111100

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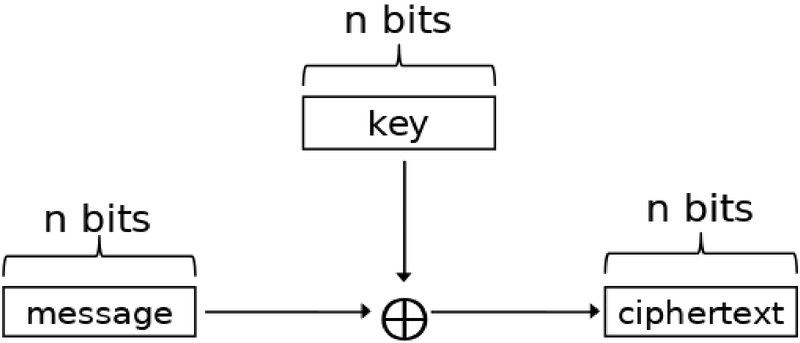
Caveat: Generating truly random bits is hard.

One-time pad



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- The OTP was Proven perfectly secret by Shannon in 1949.

Linear Cong. Generators

How Hard is it to Generate Truly Random Bits?

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Bill Okay. How does Java do it? Is it *Truly* Random?

Student Oh. Okay, you tell me– how does Java do it?

Bill I will show what Java does and why it bytes.

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Depending on A, B, x_0 this can look random... or not.

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Eve will assume that A and M are rel prime.

Example of Linear Cong. Gen

$x_0 = 21,\ A = 19,\ B = 30,\ M = 91$

$x_0 = 21$

$x_1 = 19 * 21 + 30 \pmod{91} = 65$

$x_2 = 19 * 65 + 30 \pmod{91} = 82$

$x_3 = 19 * 82 + 30 \pmod{91} = 41$

$x_4 = 19 * 41 + 30 \pmod{91} = 81$

$x_5 = 19 * 81 + 30 \pmod{91} = 22$

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Does this sequence look random? Hard to say.

Our Running Example

$x_0 = 2134,\ A = 4381,\ B = 7364,\ M = 8397.$

$$\begin{aligned} x_0 &= 2134 \text{ view as } 21, 34 \\ x_{n+1} &= 4381 x_n + 7364 \pmod{8397} \end{aligned}$$

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We will assume Eve knows that the random numbers are gen by a recurrence of the form

$$x_{i+1} = Ax_i + B \pmod{M}$$

but that Eve do not know x_0, A, B, M. Does know A, B rel prime.

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   So send 31 51 (these do not correspond to letters, thats fine).

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How to get a long random (looking?) sequence? Next slide.

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We show that this random-looking sequence is NOT that random and, if used for a psuedo-one-time-pad, can be cracked.

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If the document began with the word **secret** then encode by adding columns base 10:

Text-Letter	S	E	C	R	E	T
Text-Digits	19	05	03	18	05	20
Key–Digits	21	60	69	05	37	78
Ciphertext	30	65	62	13	32	98

Note E is coded as 65 and then later as 32. Recall that the whole point of OTP is that a letter won't always be coded the same way.

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Can we solve these? (The title **Eve Can Crack It!** gives it away!)

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Leave as an exercise how many equations.

Eve Can Crack It!—Finding M (I)

EQ1: $7648 \equiv 1865A + B \pmod{M}$
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Add EQ4' and EQ5' to get: $-36392598 \equiv 0 \pmod{M}$
Can we use this?

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Hence a SMALL number of possibilities for M.

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Two ways to find possibilities for M on next few slides.

Eve Factors to Find M

Eve factors 36392598.

$36392598 = 2 \times 3^3 \times 11 \times 197 \times 311$

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6. If use 311 and 27: $311 \times 27 = 8397$. WORKS!

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7. Leave it to you to show that using 197 does not work.
8. So $M =$ **8397**.

That Last Slide was Old-Timey

That last slide was the sort of thing people did before computers.

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In fact, today we would do something even less clever—we discuss later.

Reflect

We found $M = 8397$ is only M that works..

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We might have found **no** M works. In that case, goto next 8-sequence.

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We might have found **several** M works. In that case, do what is on the next few slides with each one.

Eve Can Crack It—Finding A

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By either brute force of cleverness we found that $M = \mathbf{8397}$.

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Reflect It is possible the inverse does not exist. Then move on to next 8-sequence. In the case at hand, the inverse exists.

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Multiply both sides of EQ4 by 1982 to get:

$$-6823 \times 1982 \equiv A \pmod{8397}$$

$$A \equiv -6823 \times 1982 \equiv \mathbf{4381} \pmod{8397}$$

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Now want to find B. Recall:

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By plugging in $M = 8397$ and $A = 4381$ we get

$$7648 \equiv 1865 * 4381 + B \pmod{8397}$$

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So..., are we done? Do we have correct A, B, M? Do we need more?

Eve Can Crack It!—Finding x_0

We have $A = 4381$, $B = 7634$, $M = 8307$ so we have

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We have $A = 4381, B = 7634, M = 8307$ so we have

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Need x_0.

4381 is rel prime to 8397 so $(4381)^{-1} \pmod{8397}$ exists.
It is 8374. Mult equation by 8374.

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$$8374x_{n+1} \equiv x_n + 6965 \pmod{8397}$$

$$x_n \equiv 8374x_{n+1} - 6965 \equiv 8374x_{n+1} + 1432$$

How will this help us?

Eve Can Crack It!—Finding x_0 (cont)

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Eve Can Crack It!—Finding x_0 (cont)

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PAKISTAN had the P on the (say) 191st spot. We know the key at 191 spot. Hence can use recurrence above to get key at 190th, 189th, ..., 0th spot.

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Are we done yet? No.

Eve Uses Is-English

Eve has x_0, A, B, M so Eve can generate the **entire** key.

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Use IS-ENGLISH.

If YES, then done.

If NO, then go to next 8-seq or next M if there was one.

Putting it All Together

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1. Input is long ciphertext T that Eve knows was coded with recurrence. Eve knows a word w that she knows appears in the text and is ≥ 8 letters. $w = w_1 \cdots w_8$ is first 8 letters.

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   2.2 Use A, B, M, x_0 to generate **entire** key. Decode **entire** text. If IS-ENGLISH=YES, DONE! Else goto next 8-let-seq.

Eve Can Factor Fast?

Eve had to factor:

$$36,392,598 = 2 \times 3^3 \times 11 \times 197 \times 311$$

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Our scenario is closer to **random** than to **Alice** .

With Modern Computers do not Need to be Clever

Recall

(1) M div 36392598, (2) M 4 digs long, (3) $7649 \le M \le 9999$.

How to find M?

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PROS and **CONS**

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This gives a small set of possibilities for M.

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Real World Versus What I Teach (I)

Paraphrase of a **Recent conversation with Zan**

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This exciting conversation continued on next slide!

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Real World versus What I Teach

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Real World versus What I Teach

Paraphrase of a **Recent conversation with Zan (cont)**

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Bill I will teach them how to crack LCG in the general case, but then comment that often M is a power of 2.

Zan Okay, that works. You are truly the master of education (NOTE: Zan did not say that, but he did call me a moron again.)

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Paraphrase of a **Recent conversation with a Student**

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Student Challenge? What challenge?

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4. The larger the parameter which we have as 7, the longer the phrase has to be.

Mersenne Twister Example with Digits

Text-Letter	P	A	K	I	S	T	A	N	B	O
Text-Digits	16	01	11	09	19	20	01	14	02	15
Cipher-text	24	66	87	47	17	45	26	96	06	11
Key	18	65	76	48	08	25	25	82	04	04
Text-Letter	R	D	E	R	S	I	N	D	I	A
Text-Digits	18	04	05	18	19	09	14	04	09	01
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Key	95	12	04	03	90	10	16	07	15	09

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Can use recurrences to find f, a, b.Will need more equations and some guesswork, but crackable!

Upshot

Any pseudo-random generator that is based on recurrences is crackable.

An Approach To Generating Random Bits

Random-number generation

1. Continually collect ‘unpredictable” data.
2. This data may be biased.
3. Correct biases in it to make it more random.
4. Called **smoothing** .

Unpredictable: Different models. Our Model: There is a $0 < p < 1$ such that each bit has

$$\Pr(1) = p, \Pr(0) = 1 - p.$$

Bits are independent. p is not known.

Smoothing via Von Neumann Technique (VN)

- Need to eliminate *bias*.
- VN technique for eliminating bias:
  - Collect two bits per output bit
    - $01 \mapsto 0$
    - $10 \mapsto 1$
    - $00, 11 \mapsto \text{skip}$
  - Note that this assumes *independence* (as well as constant bias)
  - This gives truly random bits (next slide) but takes time.

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Perfect Randomness!

How Many Random Bits Can We Expect?

Assume that $\Pr(b = 0) = p$ and $\Pr(b = 1) = 1 - p$.

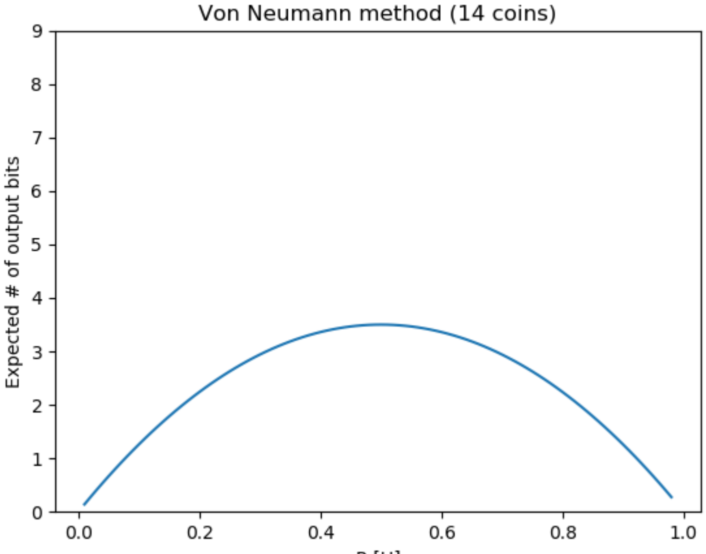
If flip 2 coins then expected numb of rand bits is

$$\Pr(01) + \Pr(10) = p(1 - p) + (1 - p)p = 2p(1 - p).$$

If flip $2n$ coins then expected number of rand bits is $2np(1 - p)$.

How Good is VN Method?

If flip 14 coins ($n = 7$) then we get the following graph:



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2. The method can be extended, called **The Elias Method.** We won't present it but will show graph on next slide.

VN vs GMS

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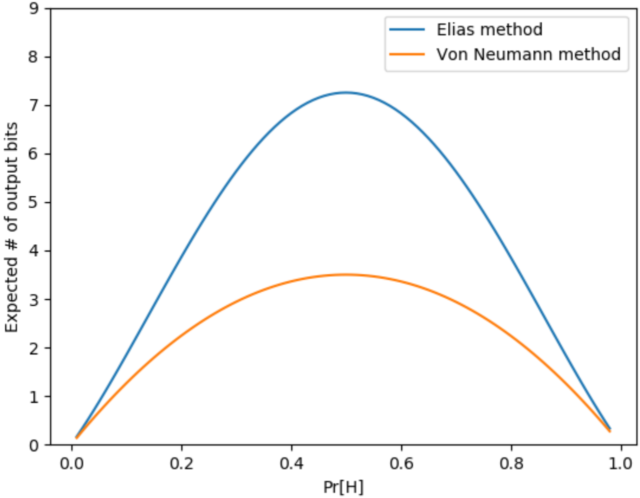

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3. So can we get truly random bits?

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