BILL, RECORD LECTURE!!!!

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Public Key Cryptography: RSA

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Quote: . . . And the ELN's **strong encryption system** has prevented the army from extracting information from seized computers, as it did with FARC.

Caveat: The article did not say what system they used. **Oh Well.**

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They Mean: Someone smarter than me can surely prove this.

The Academic Code, More Examples

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When Academics Say: It is generally believed that. . ..
They Mean: Me and my friends think. . ..

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What does RSA Stand For?

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They are the ones who came up with this cryptosystem.

Slight Variant on Fermat's Little Theorem

Recall Fermat's little Theorem

Thm If p is prime and $a \in \mathbb{N}$ then

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We will refer to both as **Fermat's Little Theorem.**

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Now do normal repeated squaring, $2\lg(27) = 10$. Can do better.
Recall its really
$\lg(27)+$ the number of 1's in the binary rep of 27.

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This last equation is the important point

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Next few slides are on the ϕ function.

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Needed Mathematics- The ϕ Function (cont)

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Recall For all m, $a^m \equiv a^{m \pmod{p-1}} \pmod{p}$.

So arithmetic in the exponents is mod $p-1$.

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Recall If a, b rel prime then $\phi(ab) = \phi(a)\phi(b)$.

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We restate and generalize.

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Restate:

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Generalize:

Fermat-Euler Theorem If $n \in \mathbb{N}$ and a is rel prime to n then

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Examples

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Now just do repeated squaring.

Bait and Switch

I got you interested in the theorem

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by telling you that it can be used to do things like

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with **much less than** $2\lg(191, 992, 194, 299, 292, 777)$ mults.

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You are thinking A&B will need to do $a^m \pmod{n}$ for **large** m.

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$$a^m \equiv a^{m \bmod \phi(n)} \pmod{n}$$

by telling you that it can be used to do things like

$$17^{191,992,194,299,292,777} \pmod{150}.$$

with **much less than** $2\lg(191,992,194,299,292,777)$ mults.
This is true! There will be some HW using it.

You are thinking A&B will need to do $a^m \pmod{n}$ for **large** m.

No. That is not what we will be doing, though I see why you would think that. Or you see why I think you would think that. Or ....

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Question Can Eve find out m?

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In examples we do in slides and HW we might not have $e \in \{R/3, \ldots, 2R/3\}$ since we want to have easy computations for educational purposes.

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Use primes:
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Alice compute $c^d \pmod{1147}$, should get back m.

What Do We Really Know about RSA

If Eve can factor then she can crack RSA.

1. Input (N, e) where $N = pq$ and e is rel prime to $R = (p-1)(q-1)$. (p, q, R are NOT part of the input.)
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Note In ugrad math classes rare to have a statement that is **UNKNOWN TO SCIENCE**. **Discuss**.

Hardness Assumption

Definition Let f be the following function:
Input $N, e, m^e \pmod{N}$ (know $N = pq$ but don't know p, q).

Outputs m.

Hardness assumption (HA) f is hard to compute.
One can show, assuming HA that RSA is hard to crack. But this proof will depend on a model of security. See caveats about this on similar DH slides (bribery, timing attacks, Maginot Line).

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Item 4 is current state with some caveats: Do Alice and Bob use it properly? Do they have large enough parameters? What is Eve's computing power?

RSA has NY,NY Problem. Will Fix

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Plain RSA is never used and should never be used!

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Alice and Bob pick L_1 and L_2 such that $\lg N = L_1 + L_2$.
To send $m \in \{0,1\}^{L_2}$ pick random $r \in \{0,1\}^{L_1}$.
When Alice gets rm she will know that m is the last L_2 bits.

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Important If later Bob wants to send 100 again he will choose a DIFFERENT random 3 bits so Eve won't know he sent the same message.

RSA has Another Problem

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(1) will confuse Alice (2) Sealed Bid Scenario.

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4. Name BLAH-1.5 is hint that it's not final version.

Other Public Key Systems

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1. The problems above make it not practical.
2. The problems above could have been gotten around but RSA just got to the market faster.

RSA Summary

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How Important Is Public Key?

Used Everywhere

Public key is mostly used for giving out keys to be used for classical systems.
This makes the following work:

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5. Military – though less is known about this.

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3. There are now several Public Key Systems based on **other** hardness assumptions. See next slide.

Public Key Not Based on Factoring (cont)

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`https://www.scribd.com/document/474476570/PQC-Overview-Aug-2020-NIST`

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