The Vigenère Cipher

**Key:** \( k = (k_1, k_2, \ldots, k_n) \).

**Encrypt** (all arithmetic is mod 26)

\[
Enc(m_1, m_2, \ldots, m_N) = m_1 + k_1, m_2 + k_2, \ldots, m_n + k_n, m_{n+1} + k_1, m_{n+2} + k_2, \ldots, m_{n+n} + k_n, \ldots
\]

**Decrypt** Decryption just reverses the process
Three Kinds of Vigenère Ciphers

1. Standard Vig: Use a longish-sentence. Key is Sentence.
3. one-time pad: Key is random gen sequence.
Cracking Vig cipher: Step One-find Keylength

One have keylength can crack, so okay to try many of them. Two ways to guess keylengths:

1. Spot (say) a 4-letter sequence that appears 5 times and use differences of appeared to narrow down key length.
2. Try all keylengths of length 1,2,3,... until you hit it.
Cracking the Vig cipher: Step Two-Freq Anal

After Step One we have the key length $L$. Note:

1. Separate text $T$ into $L$ streams depending on position mod $L$.
2. For each stream try every shift and use Is English to determine which shift is correct.
3. You now know all shifts for all positions. Decrypt!
Cracking the Vig cipher: Step Two-Freq Anal

After Step One we have the key length $L$. Note:

- Every $L^{th}$ character is “encrypted” using the same shift.

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1. Separate text $T$ into $L$ streams depending on position mod $L$.
2. For each stream try every shift and use *Is English* to determine which shift is correct.
3. You now know all shifts for all positions. Decrypt!
If the key was **Corn Flake**

You would get a key of length 9. We want **More**.
Getting More Out of Your Phrase

If the key was

Corn Flake

You would get a key of length 9. We want More.

Corn is 4 letters long. Flake is 5 letters long.
We form a key of length $LCM(4, 5) = 20$. (Won’t fit on line! Oh Well.)

\[
\begin{array}{cccccccccc}
C & O & R & N & C & O & R & N & C & O \\
F & L & A & K & E & F & L & A & K & E \\
7 & 25 & 17 & 23 & 6 & 19 & 2 & 13 & 12 & 18 \\
\end{array}
\]

ADD it up to get new 20-long key.
A student said:

*Let’s use Vig cipher with a book for the key*

Is it a good idea? Discuss
A student said:

*Let’s use Vig cipher with a book for the key*  
Is it a good idea? **Discuss**

1. Before modern computer era: YES.  
2. Now. NO.
How to Crack the Vig Book Cipher

Eve sees a $d$. (Recall that $d = 3$.) What does Eve know? Discuss
How to Crack the Vig Book Cipher

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Eve knows that $(\text{First Letter in Key}) + (\text{First Letter in Text}) = 3$. Hence the following are the only possibilities for $(\text{Letter in Key, Letter in Text})$ are:

$$(a, d), (z, e), (y, f), (w, g), \ldots, (b, c)$$

Only 26 possibilities. What of it? Discuss
How to Crack the Vig Book Cipher

Eve sees a $d$. (Recall that $d = 3$.) What does Eve know? Discuss

Eve knows that (First Letter in Key) + (First Letter in Text) = 3. Hence the following are the only possibilities for (Letter in Key, Letter in Text) are:

$$(a, d), (z, e), (y, f), (w, g), \ldots, (b, c)$$

Only 26 possibilities. What of it? Discuss

Some of the pairs are more likely than others.

1. **Both** the key and the text are in English.
2. $(z, e)$: Hmm, $z$ is unlikely but $e$ is likely.
3. $(a, d)$: Hmm, seems more likely than $(z, e)$.
4. Can rank which are more likely (e.g., add or mult the freqs).
5. Can then use adjacent letters and freq of adjacent pairs, and rank them.
One-Time Pad

Let $\mathbb{M} = \{0, 1\}^n$, the set of all messages.

- $\text{Gen}$: choose a uniform key $k \in \{0, 1\}^n$.
- $\text{Enc}_k(m) = k \oplus m$.
- $\text{Dec}_k(c) = k \oplus c$.

Correctness: $\text{Dec}_k(\text{Enc}_k(m)) = k \oplus (k \oplus m) = (k \oplus k) \oplus m = m$. 
One-Time Pad

- Let \( \mathcal{M} = \{0, 1\}^n \), the set of all messages.
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One-Time Pad

1. **PRO** $⊕$ is FAST!
2. **CON** If Key is $N$ bits long can only send $N$ bits.
3. **PRO** Uncrackable if use truly random bits.
4. **CON** Hard to get truly random bits.
Ways to Get Random-Looking Bits

1. **Linear Cong Gen** Pick $x_0, A, B, M$ at random and then use:
   
   $x_0$
   
   $x_{i+1} = Ax_i + B \pmod{M}$
   
   We summarize how to crack VERY BRIEFLY after this slide.

2. **Merseen Twister** Also a recurrence, also crackable.

3. **VN method** if can generate bits with $\text{prob}(0) = p$, $\text{prob}(1) = 1 - p$ can use to get truly random bits without knowing $p$.
   
   Takes a long time to get the bits.
   
   Generating bits with prob $p, 1 - p$, still hard.

4. **Elias method** Did not do in class, but
   
   Better than VN for time to get bits.
   
   Generating bits with prob $p, 1 - p$, still hard.

5. **We will see better methods later in the course.**
1. Have some word or phrase that you think is there. E.g., **PAKISTAN**. Say its 8 letters.

2. For EVERY 8-letter block (until you succeed) do the thought experiment: What if its **PAKISTAN**?
   2.1 Based on that guess find equations that relate \( A, B, M \).
   2.2 Try to solve those equations. If no solution goto next block-of-8.
   2.3 There is \( \geq 1 \) solution \((A, B, M)\). Use it to find \( x_0 \) and the entire plaintext \( T \).
   2.4 Test if \( T \) IS-English. If so then DONE. If not then goto next block-of-8.
The Matrix Cipher

**Def** Matrix Cipher. Pick $M$ an $n \times n$ invertible over mod 26 matrix.

1. Encrypt via $xy \rightarrow M(xy)$.
2. Decrypt via $xy \rightarrow M^{-1}(xy)$.

**Encode:** Break text $T$ into blocks of 2, apply $M$ to each pair.

**Decode:** Do the same only with $M^{-1}$. 

The Matrix Cipher: Good and Bad

**Good News:**
1. Can test if $M^{-1}$ exists, and if so find it, easily.
2. $M$ small, so Key small.
3. Applying $M$ or $M^{-1}$ to a vector is easy computationally.
4. Not clear if Eve can crack using Ciphertext Only Attack.
5. If $n$ is large enough Eve cannot use brute force, but see next slide.

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The Matrix Cipher: Good and Bad

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**Bad News:**

1. Input $T$, a coded text.
2. For EVERY $8 \times 8$ invertible matrix $M$ over mod 26,
   2.1 Decode $T$ into $T'$ using $M$.
   2.2 IF LOOKS-LIKE-ENGLISH($T'$) = YES then STOP and output $T'$, else goto next matrix $M$.

Takes roughly $26^{64}$ steps.
Can Crack in $8 \times 26^8$

The attack in the last slide went through every Matrix.
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Better Idea: We take life one row at a time.
Can Crack in $8 \times 26^8$

The attack in the last slide went through every Matrix. **Better Idea:** We take life **one row at a time**.
**Example:** $3 \times 3$ matrix cipher. Decode Matrix $M$.

$$T = t_1 t_2 \cdots t_N$$

each $t_i$ is 3-long

### Example

\[
\begin{bmatrix}
1 & 1 & 7 \\
\ast & \ast & \ast \\
\ast & \ast & \ast \\
\end{bmatrix}
\]
Can Crack in $8 \times 26^8$

The attack in the last slide went through every Matrix.

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**Example:** $3 \times 3$ matrix cipher. Decode Matrix $M$.

$$T = t_1 t_2 \cdots t_N \text{ each } t_i \text{ is 3-long}$$

Guess the first row of $M$. Say:

$$
\begin{pmatrix}
1 & 1 & 7 \\
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Guess the first row of $M$. Say:

$$
\begin{pmatrix}
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* & * & * \\
* & * & *
\end{pmatrix}
$$

Let $M t_i = m_i$. Then $(1, 1, 7) \cdot t_i = m_i^1$ is first letter of $m_i$.

$$(m_1^1, m_2^1, m_3^1, \ldots, m_N^1)$$

is every third letter. Can do IS-ENGLISH on it.
Can Crack in $8 \times 26^8$

Eve knows that Alice and Bob decode with $8 \times 8$ Matrix $M$. Ciphertext is

$$T = t_1 t_2 \cdots t_N \quad t_i = t_i^1 \cdots t_i^8$$

For $i = 1$ to 8

For all $r \in \mathbb{Z}_{26}^8$ (guess that $r$ is $i$th row of $B$).

$T' = (r \cdot t_1, \ldots, r \cdot t_N)$ (Is every 8th letter.)

IF IS-ENGLISH($T'$) = YES then $r_i = r$ and goto next $i$. Else goto the next $r$.

$M$ is

$$\begin{pmatrix}
\cdots & \cdots & \cdots & \cdots \\
\vdots & \vdots & \vdots & \vdots \\
r_1 & \cdots & r_n \\
\vdots & \vdots & \vdots & \vdots \\
\cdots & \cdots & \cdots & \cdots
\end{pmatrix}$$

Takes $8 \times 26^8$ steps.
More General $n$

If $M$ is $n \times n$ matrix.
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Brute force takes $O(26^{n^2})$. 
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Brute force takes $O(26^{n^2})$.

The row-by-row method takes $O(n26^n)$.
Important Lesson

Assume: $26^{64}$ time is big enough to thwart Eve.

1. If we think that best Eve can do is $O(26^n^2)$ then we take $n = 8$, so Eve needs $O(26^{64})$.

2. If we think that best Eve can do is $O(n26^n)$ then we take $n = 80$, so Eve needs $O(80 \times 26^{80})$.

The $O(n \times 26^n)$ cracking does not show that Matrix Cipher is insecure, but it still is very important: Alice and Bob must increase their parameters. That is already a win since it makes life harder for Alice and Bob.
The History of Cryptography in One Slide

1. Alice and Bob come up with a Crypto system (e.g., Matrix Cipher with $n = 8$).
2. Alice and Bob think its uncrackable and have a “proof” that it is uncrackable (e.g., Eve HAS to go through all $26^{64}$ matrices).
3. Eve Cracks it. (The trick above—only about $8 \times 26^8$.)
4. Lather, Rinse, Repeat.

Above attack on Matrix Cipher is a microcosm of this history. Proofs rely on limiting what Eve can do, and hence do not work if Eve does something else.
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Cracking Matrix Cipher With Known Ciphertext Attack

Example using $2 \times 2$ Matrix Cipher.
Eve learns that (13,24) encrypts to (3, 9). Hence:

$$
\begin{pmatrix}
a & b \\
c & d
\end{pmatrix}
\begin{pmatrix}
13 \\
24
\end{pmatrix}
= 
\begin{pmatrix}
3 \\
9
\end{pmatrix}
$$

So $13a + 24b = 3$ and $13c + 24d = 9$. Two linear equations, Four variables.

If Eve learns one more 2-letter message decoding then she will have Four linear equations, Four variables which she can solve!
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Cracking Matrix Cipher With Known Ciphertext Attack

Example using 2 × 2 Matrix Cipher.
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\end{pmatrix}
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24 \\
\end{pmatrix}
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So

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**Two linear equations, Four variables**

If Eve learns one more 2-letter message decoding then she will have
**Four linear equations, Four variables**
which she can solve! Yeah? Boo? Depends whose side you are on.
Upshot

1. Matrix Cipher with ciphertext only might be hard to crack.
2. Matrix Cipher where Eve has access to prior messages is easy to crack.
3. We need to better refine our notion of attack.
4. We will do this in the next set of slides.
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Ciphertext and Plaintext

**Plaintext** The message Alice really wants to send to Bob, e.g. *Meet me after class.*
**Ciphertext and Plaintext**

**Plaintext** The message Alice really wants to send to Bob, e.g.

   Meet me after class.

**Ciphertext** What Alice sends Bob. The hope is that if Eve sees it she will **not** learn the plaintext. E.g.

   PHHWP HDIWH UFODV V
Types of Attacks

We will describe several different types of attacks Eve can use. They depend on:

1. What information Eve has access to.
2. What computing power Eve has.
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1. What information Eve has access to.
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Eve’s goal is to find out something about the plaintext she did not already know.
Ciphertext Only Attack (COA)

All Eve has is the ciphertext.
Ciphertext Only Attack (COA)

Ciphertext Only Attacks (COA) All Eve has is the ciphertext.
Eve cracked shift, affine, general sub, Vig with a COA.
Known Plaintext Attack (KPA)

**Known Plaintext Attack (KPA)** Eve knows the plaintext for some of the ciphertext.

1. If yesterday the message **ABC DEFG** was where the spy would be, and today Eve found the spy in **New York**, then **ABC DEFG** decodes to **New York**.
2. **WWII History** A German soldier in an area where nothing was happening sent nothing to report every day.
3. **Guess a word that you think appears in the document.** For Linear-Cong-Gen our example was **Pakistan**.
4. **More WWII History** Turing and his gang cracked the German Enigma Code, guessing that **ein** (German for one) would be in messages.
Known Plaintext Attack (KPA) Eve knows the plaintext for some of the ciphertext.

Eve can crack Matrix and Linear-Cong-Gen ciphers with a KPA.
Known Plaintext Attack (KPA)  

**Known Plaintext Attack (KPA)**  Eve knows the plaintext for some of the ciphertext.

Eve can crack Matrix and Linear-Cong-Gen ciphers with a KPA. How can Eve obtain plaintext for some of the ciphertext?
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Chosen Plaintext Attack (CPA)

**Chosen Plaintext Attack (CPA)** Eve tricks Alice into encoding a particular plaintext.

---

1. **WWII History**
   - America thought that the Japanese code for Midway was **AF**. So the Americans send the message to a ship Midway is low on supplies.
   - The Americans observe that the next Japanese message has **AF** in it, so their suspicions are confirmed.

2. **WWII History**
   - England put mines in places that the Germans had no abbreviations for. The Germans cleared those mines and send **NAME OF PLACE, all clear**, transmitted in code.
**Chosen Plaintext Attack (CPA)**

Eve tricks Alice into encoding a particular plaintext. Later in the course we will see a CPA attack on RSA.
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2. **WWII History** England put mines in places that the Germans had no abbreviations for. The Germans cleared those mines and send **NAME OF PLACE, all clear**, transmitted in code.
Chosen Ciphertext Attack (CCA) Eve tricks Alice into decoding a particular ciphertext.
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Dictionary Attack

Dictionary Attack Many variants, we give two:

1. Have database of X decodes to Y and pattern match.
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- Password
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- qwerty
- 12345
- 12345678
- letmein
- 1234567
- football iloveyou
- admin welcome monkey login abc
- 123
- starwars
- 123123
- dragon passwOrd master
- hello freedom whatever qazwsx trusno
- qwerty: 1st 6 letters on 3rd line of a keyboard.
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Brute Force Attacks (BFA)

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1. The key space is small enough for Eve's computing power.
2. Clever way to reduce key space before you do BFA.

Easy to thwart
Use a bigger key space!
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Use how much time or power a cryptosystem is using to figure out information about the key and narrow down the keyspace.
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Look up the Maginot Line.
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3. Look up the Maginot Line.
The Playfair Cipher: The Key

We use $\Sigma = \{a, \ldots, z\} - \{j\}$. Need a square. If need to use $j$ use an $i$.

**Key** is a word or phrase. Delete all repeats from it. We will use **Bill Gasarch** which becomes BILGASRCH. Use the key to start a $5 \times 5$ array of all of the letters

<table>
<thead>
<tr>
<th>B</th>
<th>I</th>
<th>L</th>
<th>G</th>
<th>A</th>
</tr>
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<tbody>
<tr>
<td>S</td>
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<td>C</td>
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<td>D</td>
</tr>
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<td>V</td>
<td>W</td>
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Given a pair, what do you map it to? Start by finding the pair in the grid.

1) If the pair are NOT in the same row or column then look at rectangle formed and take other corners. EXAMPLE: Map RA:

<table>
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<th>B</th>
<th>I</th>
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RA maps to **ID**.
The Playfair Cipher: The Other Cases

We skip them for the review.
The Rail Fence Cipher

October 12, 2020
Key is 3. Message is **Marina is a TA**.
Write it in three rows as such:
```
M N A
A I A S T
R I A
```
Write each row: **MNAIAASTRIA**
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In current case of 3 rows and message of length 11 we did
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R I A"
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In current case of 3 rows and message of length 11 we did

First list out the letters in positions \( \equiv 1 \pmod{4} \).
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Write it in three rows as such:

\[
\begin{array}{ccc}
M & N & A \\
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R & I & A \\
\end{array}
\]

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In current case of 3 rows and message of length 11 we did
First list out the letters in positions \( \equiv 1 \pmod{4} \).
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Leave as an exercise what happens if \( k \) rows, \( n \) letter message.
The Autokey Cipher

October 12, 2020
IDEA: Use the plaintext as a Key after a start.
The AutoKey Cipher: A Variant of Vig

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1. There is a key, a short word or phrase. We’ll use Metz.

2. Metz is (12, 4, 19, 25). We shift the first letter by 12, the second by 4, the third by 19, the fourth by 25.

3. After first four use plaintext lagged by 4.

Example Key is Metz and I want to encode Joe Biden is running. So Key is metzjoebidenisrunning

1. Encode (j, o, e, b) by shifting by (12, 4, 19, 25).

2. Encode (i, d, e, n, i, s, r, u, n, i, n, g) by the shift induced by (j, o, e, b, i, d, e, n, i, s, r, u, n, i, n, g).

To Decode will need to do this four letters at a time.
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**PROS:** The techniques for cracking Vig do not work.

**PROS:** If Eve does not know you are using it, seems uncrackable.

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**Question:** How would you crack it?
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Similar to Book Cipher in that the key and the message are both in English so can use freq somewhat.

If guess the key word then rest is EASY!
(Another) Book Cipher

The term **Book Cipher** was used both for the Vig cipher where key is a book and for what I will discuss now.
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**Def** Book Cipher:

1. Alice and Bob agree on a book to be the key.
2. To send a message Alice specifies, for each word,
   - A page number. E.g., Page 19.
   - A line number. E.g., Line 24 (On Page 19).
3. Alice will try to use different triples for the same word.
4. Bob has same book so can decode.

**Security**

Known to be crackable, but won't go into that here.
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A Problem with MOST of our Ciphers/Terminology

1. Most of our ciphers are deterministic so always code $m$ the same way. This leaks information.
2. One-Time Pad and Book Ciphers avoid this, but have very long keys.
3. The problem of the same message leading to the same ciphertext is called **The NY,NY Problem.**
Randomized Shift Key is a function $f : S \rightarrow S$.

1. To send message $(m_1, \ldots, m_L)$ (each $m_i$ is a character):
   1.1 Pick random $r_1, \ldots, r_L \in S$.
   1.2 Send $((r_1; m_1 + f(r_1)), \ldots, (r_L; m_L + f(r_L)))$. 
How to Fix This Without a Long Key

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2. To decode message $((r_1; c_1), \ldots, (r_L; c_L))$:
   2.1 Find $(c_1 - f(r_1), \ldots, c_L - f(r_L))$. 
Cracking Randomized Shift

With a long text Rand Shift \textbf{is} crackable. If $N$ is long and Eve sees:

$$(r_1; \sigma_1)(r_2; \sigma_2) \cdots (r_N; \sigma_N).$$

1. If $N$ is long enough we will see EVERY $r$ MANY times.
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2. From our study of Vig we know that taking every $m$th letter in a text has the same distribution of letters as a normal text.
3. It turns out that taking a random set of letters also has the same distribution as a normal text.
Cracking Randomized Shift Final Algorithm

1. Input \((r_1, \sigma_1)(r_2, \sigma_2) \cdots (r_N, \sigma_N)\)

2. For each \(r \in \{1, \ldots, 26\} \):
   2.1 Look at the spots \((r, \sigma_1), (r, \sigma_2) \cdots (r, \sigma_L)\).
   2.2 All of these \(\sigma_i\)'s used same shift.
   2.3 Guess each shift and use IS-ENGLISH to find out which shift is correct.

3. We now have the mapping of \(r\)'s to shifts. \(r\) maps to shift \(s_r\).

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4. Can use the $s_r$’s to decode entire message.
Cracking Randomized Shift Final Algorithm

1. Input \((r_1; \sigma_1)(r_2; \sigma_2) \cdots (r_N; \sigma_N)\)

2. For each \(r \in \{1, \ldots, 26\}\):
   2.1 Look at the spots \((r, \sigma), so\)
   \((r, \sigma_1) \cdots (r, \sigma_2) \cdots (r, \sigma_L)\).
   2.2 All of these \(\sigma_i\)'s used same shift.
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