BILL START THE RECORDING
FILL OUT YOUR COURSE EVALS FOR ALL YOUR COURSES

You got an email asking you to fill out your course evals for all of your courses.

1. Teachers read them and use it to help their teaching. Especially the comments.

2. The teaching evaluation committee reads them to help the teachers with their weak spots. (I was the originator and chair of the Teaching Eval Committee for 12 years so I have seen this in action. I have also been frustrated with courses with not-that-many evals filled out!) Side Note: Nobody should be in any admin position for more than 5 years!

3. These evals are used in the promotion process (Tenure, Senior lecturer, others). It is our hope that because the Teaching Eval Comm helps people become better teachers, there is NO bad teaching so this is not an obstacle for promotion.

4. And you can help us! By filling out the forms!
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Threshold Secret Sharing: Length of Shares
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We will see that we can always get length $n + 1$. 

Length of Shares: \( n \) or \( n + 1 \)

We do secret sharing with \( |s| = n \).
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1. Using the following theorem we can always to secret sharing with shares of length \( n + 1 \):

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\text{For all } x \text{ there is a prime } p \text{ such that } x \leq p \leq 2x.
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Too teach you field theory so you can save 1 bit seems like too much sugar for a cent
That is either an old-timey saying or a password from the NSA.
Can Shares be SHORTER than Secret?

1. If we use Fields, we have size-of-shares EQUALS size-of-secret.
2. If we use Mod \( p \) with \( p \) prime, we have size-of-shares EQUALS size-of-secret (+1).

Can Zelda Secret Share with shares SHORTER than the secret?

1. YES and this is known.
2. NO and this is known.
3. YES but needs a hardness assumption.
4. UNKNOWN TO SCIENCE!

VOTE

Answer NO
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Assume there is a (4, 5) Secret Sharing Scheme where Zelda shares a secret of length 7.
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(This proof will assume NOTHING about the scheme.)
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Before the protocol begins there are $2^7 = 128$ possibilities for the secret.
Assume that $A_5$ gets a share of length 6. We show that the scheme is NOT info-theoretic secure.
Example of Why Can’t Have Short Shares, Cont

If $A_1, A_2, A_3, A_5$ got together they learn secret, since it’s a $(4, 5)$ scheme.
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If $A_1, A_2, A_3, A_5$ got together they learn secret, since it’s a $(4, 5)$ scheme.
We show that $A_1, A_2, A_3$ can learn SOMETHING about the secret.
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$CAND = \emptyset$. $CAND$ will be set of Candidates for $s$. 
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For $x \in \{0, 1\}^6$ (go through ALL shares $A_5$ could have)
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Secret is in $CAND$. $|CAND| = 2^6 < 2^7$. 
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So $A_1, A_2, A_3$ have eliminated many strings from being the secret $s$. 

That is INFORMATION!!!! On the HW you will do more examples and perhaps generalize to show can NEVER have shorter shares.
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Are Shorter Shares Ever Possible?

If we demand info-security then everyone gets a share \( \geq n \).
What if we only demand comp-security?

VOTE
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If we demand info-security then everyone gets a share $\geq n$. What if we only demand comp-security?

**VOTE**

1. Can get shares $< \beta n$ with a hardness assumption.
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Can get shares $< \beta n$ with a hardness assumption.
Will do that later.
Generalize The Problem

Our problem: Player $A_1, \ldots, A_m$, secret $s$. 

1. If at least $t$ of them get together they can find $s$.
2. If at most $t-1$ of them get together they cannot find $s$.

That is not quite right. Why?

1. If at least $t$ of them get together they can find $s$.
2. If at most $t-1$ of them get together they cannot find $s$.

We want to generalize and look at other subsets.

Example

1. If an even number of players get together can find $s$.
2. If an odd number of players get together can't find $s$.

Try to find a scheme for this secret sharing problem.

You've Been Punked!

$A_1, A_2$ CAN find $s$ but $A_1, A_2, A_3$ CANNOT. Thats Stupid!
Generalize The Problem

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What is it about Threshold?

1. If $\geq t$ of them get together they can find out secret.
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Let's rephrase that so we can generalize:

$X$ is the set of all subsets of $\{A_1, \ldots, A_m\}$ with $\geq t$ players.

1. If $Y \in X$ then the players in $Y$ can find $s$.
2. If $Y \not\in X$ then the players in $Y$ cannot find $s$.

This question makes sense. What is it about $X$ that makes it make sense?

$X$ is closed under superset:

If $Y \in X$ and $Y \subseteq Z$ then $Z \in X$. 
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1. If $Y \in \mathcal{X}$ then the players in $Y$ can find $s$. 
What is it about Threshold?

1. If $\geq t$ of them get together they can find out secret.
2. If $\leq t - 1$ of them get together they cannot find out secret.

Let’s rephrase that so we can generalize:
\[ \mathcal{X} \] is the set of all subsets of \( \{A_1, \ldots, A_m\} \) with $\geq t$ players.

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2. If $Y \notin \mathcal{X}$ then the players in $Y$ cannot find $s$.

This question makes sense. What is it about $\mathcal{X}$ that makes it make sense?
What is it about Threshold?

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This question makes sense. What is it about $X$ that makes it make sense?
$X$ is closed under superset:
If $Y \in X$ and $Y \subseteq Z$ then $Z \in X$. 
**Def** An **Access Structure** is a set of subset of \( \{A_1, \ldots, A_m\} \) closed under superset.
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**Access Structures**

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**Access Structures**

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2. \((t, m)\)-Threshold is an Access structure. The poly method gives a Secret Sharing scheme where all the shares are the same length as the secret.

**Def** A secret sharing scheme is **ideal** if all shares same size as secret.
DISJOINT-OR of AND: Ideal Sec Sharing Protocol

Want that a group can find the secret if either it has

1. at least 2 of $A_1$, $A_2$, $A_3$, OR
2. at least 4 of $B_1$, $B_2$, $B_3$, $B_4$, $B_5$, $B_6$, $B_7$.

How can Zelda do this?

1. Zelda does (2, 3) secret sharing with $A_1$, $A_2$, $A_3$.
2. Zelda does (4, 7) secret sharing with $B_1$, $B_2$, $B_3$, $B_4$, $B_5$, $B_6$, $B_7$.

To generalize this we need a better notation.
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To generalize this we need a better notation.
Notation for Threshold

Let $TH_A(t, m)$ be the Boolean Formula that represents at least $t$ out of $m$ of the $A_i$’s.
Notation for Threshold

Let $TH_A(t, m)$ be the Boolean Formula that represents at least $t$ out of $m$ of the $A_i$'s.

**Example** $TH_A(2, 4)$ is
At least 2 of $A_1, A_2, A_3, A_4$.
Notation for Threshold

Let $TH_A(t, m)$ be the Boolean Formula that represents at least $t$ out of $m$ of the $A_i$’s.

**Example** $TH_A(2, 4)$ is
At least 2 of $A_1, A_2, A_3, A_4$.

**Example** $TH_B(3, 6)$ is
At least 3 of $B_1, \ldots, B_6$. 
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**Note** $TH_A(t, m)$ has ideal secret sharing.
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**Notation** $TH_A(t_1, m_1) \lor TH_B(t_2, m_2)$ means that:
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1. $\geq t_1$ $A_1, \ldots, A_{m_1}$ can learn the secret.
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1. $\geq t_1$ $A_1, \ldots, A_{m_1}$ can learn the secret.
2. $\geq t_2$ $B_1, \ldots, B_{m_2}$ can learn the secret.
3. No other group can learn the secret (e.g., $A_1, A_2, B_1$ cannot)
Disjoint OR of $TH_A(t, m)$’s: Ideal Sec Sharing

There is Ideal Secret Sharing for $TH_A(t_1, m_1) \lor \cdots \lor TH_Z(t_{26}, m_{26})$
Disjoint OR of $TH_A(t, m)$’s: Ideal Sec Sharing

There is Ideal Secret Sharing for $TH_A(t_1, m_1) \lor \cdots \lor TH_Z(t_{26}, m_{26})$

1. Zelda and the $A_1, \ldots, A_{m_1}$ do $(t_1, m_1)$ secret sharing.
Disjoint OR of $TH_A(t, m)$’s: Ideal Sec Sharing

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1. Zelda and the $A_1, \ldots, A_{m_1}$ do $(t_1, m_1)$ secret sharing.

2. : 
Disjoint OR of $TH_A(t, m)$’s: Ideal Sec Sharing

There is Ideal Secret Sharing for $TH_A(t_1, m_1) \lor \cdots \lor TH_Z(t_{26}, m_{26})$

1. Zelda and the $A_1, \ldots, A_{m_1}$ do $(t_1, m_1)$ secret sharing.

2. :

3. Zelda and the $Z_1, \ldots, Z_{m_{26}}$ do $(t_{26}, m_{26})$ secret sharing.

Note We now have a large set of non-threshold scenarios that have ideal secret sharing.
AND of $TH_A(t, m)$s: An Example

We want that if $\geq 2$ of $A_1, A_2, A_3, A_4$ AND $\geq 4$ of $B_1, \ldots, B_7$ get together than they can learn the secret, but no other groups can. Think about it.
**AND of $TH_A(t, m)$s: An Example**

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1. Zelda has secret $s$, $|s| = n$. 
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1. Zelda has secret $s$, $|s| = n$.
2. Zelda generates random $r \in \{0, 1\}^n$. 

AND of $TH_A(t, m)$s: An Example

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1. Zelda has secret $s$, $|s| = n$.
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3. Zelda does $(2, 4)$ secret sharing of $r$ with $A_1, A_2, A_3, A_4$. 


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1. Zelda has secret $s$, $|s| = n$.
2. Zelda generates random $r \in \{0, 1\}^n$.
3. Zelda does $(2, 4)$ secret sharing of $r$ with $A_1, A_2, A_3, A_4$.
4. Zelda does $(4, 7)$ secret sharing of $r \oplus s$ with $B_1, \ldots, B_7$. 
AND of $TH_A(t,m)$s: An Example

We want that if $\geq 2$ of $A_1, A_2, A_3, A_4$ AND $\geq 4$ of $B_1, \ldots, B_7$ get together than they can learn the secret, but no other groups can. Think about it.

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5. If $\geq 2$ of $A_i$’s get together they can find $r$. 
AND of $TH_A(t, m)$s: An Example

We want that if $\geq 2$ of $A_1, A_2, A_3, A_4$ AND $\geq 4$ of $B_1, \ldots, B_7$ get together than they can learn the secret, but no other groups can. Think about it.

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2. Zelda generates random $r \in \{0, 1\}^n$.
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4. Zelda does $(4, 7)$ secret sharing of $r \oplus s$ with $B_1, \ldots, B_7$.
5. If $\geq 2$ of $A_i$’s get together they can find $r$.
   If $\geq 4$ of $B_i$’s get together they can find $r \oplus s$. 
AND of $TH_A(t, m)$s: An Example

We want that if $\geq 2$ of $A_1, A_2, A_3, A_4$ AND $\geq 4$ of $B_1, \ldots, B_7$ get together than they can learn the secret, but no other groups can. Think about it.

1. Zelda has secret $s$, $|s| = n$.
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4. Zelda does $(4, 7)$ secret sharing of $r \oplus s$ with $B_1, \ldots, B_7$.
5. If $\geq 2$ of $A_i$’s get together they can find $r$.
   If $\geq 4$ of $B_i$’s get together they can find $r \oplus s$.
So if they all get together they can find

$$r \oplus (r \oplus s) = s$$
AND of $TH_A(t, m)$s: General

$TH_A(t_1, m_1) \land \cdots \land TH_Z(t_{26}, m_{26})$ can do secret sharing.
AND of $TH_A(t, m)$s: General

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AND of $TH_A(t, m)$s: General

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1. Zelda has secret $s$, $|s| = n$.
2. Zelda generates random $r_1, \ldots, r_{25} \in \{0, 1\}^n$. 
AND of $TH_A(t, m)$s: General

$TH_A(t_1, m_1) \land \cdots \land TH_Z(t_{26}, m_{26})$ can do secret sharing.

1. Zelda has secret $s$, $|s| = n$.
2. Zelda generates random $r_1, \ldots, r_{25} \in \{0, 1\}^n$.
3. Zelda does $(t_1, m_1)$ secret sharing of $r_1$ with $A_i$’s.
AND of $TH_A(t, m)$s: General

$TH_A(t_1, m_1) \land \cdots \land TH_Z(t_{26}, m_{26})$ can do secret sharing.

1. Zelda has secret $s$, $|s| = n$.
2. Zelda generates random $r_1, \ldots, r_{25} \in \{0, 1\}^n$.
3. Zelda does $(t_1, m_1)$ secret sharing of $r_1$ with $A_i$'s.
4. $\vdots$
AND of $TH_A(t, m)$s: General

$TH_A(t_1, m_1) \land \cdots \land TH_Z(t_{26}, m_{26})$ can do secret sharing.

1. Zelda has secret $s$, $|s| = n$.
2. Zelda generates random $r_1, \ldots, r_{25} \in \{0, 1\}^n$.
3. Zelda does $(t_1, m_1)$ secret sharing of $r_1$ with $A_i$'s.
4. :
5. Zelda does $(t_{25}, m_{25})$ secret sharing of $r_{25}$ with $Y_i$'s.
AND of $TH_A(t, m)$s: General

$TH_A(t_1, m_1) \land \cdots \land TH_Z(t_{26}, m_{26})$ can do secret sharing.

1. Zelda has secret $s$, $|s| = n$.
2. Zelda generates random $r_1, \ldots, r_{25} \in \{0, 1\}^n$.
3. Zelda does $(t_1, m_1)$ secret sharing of $r_1$ with $A_i$'s.
4. 
5. Zelda does $(t_{25}, m_{25})$ secret sharing of $r_{25}$ with $Y_i$'s.
6. Zelda does $(t_{26}, m_{26})$ secret sharing of $r_1 \oplus \cdots \oplus r_{25} \oplus s$ with $Z_i$'s.
**AND of $TH_A(t, m)$s: General**

$TH_A(t_1, m_1) \land \cdots \land TH_Z(t_{26}, m_{26})$ can do secret sharing.

1. Zelda has secret $s$, $|s| = n$.
2. Zelda generates random $r_1, \ldots, r_{25} \in \{0, 1\}^n$.
3. Zelda does $(t_1, m_1)$ secret sharing of $r_1$ with $A_i$’s.
4. 
5. Zelda does $(t_{25}, m_{25})$ secret sharing of $r_{25}$ with $Y_i$’s.
6. Zelda does $(t_{26}, m_{26})$ secret sharing of $r_1 \oplus \cdots \oplus r_{25} \oplus s$ with $Z_i$’s.
7. If $\geq t_1$ of $A_i$’s get together they can find $r_1$. If $\geq t_2$ of $B_i$’s get together they can find $r_2$. \cdots If $\geq t_{25}$ of $Y_i$’s get together they can find $r_{25}$. If $\geq t_{26}$ of $Z_i$’s get together they can find $r_1 \oplus \cdots \oplus r_{25} \oplus s$. So if they call get together they can find

$$r_1 \oplus \cdots \oplus r_{25} \oplus (r_1 \oplus \cdots \oplus r_{25} \oplus s) = s$$
**Definition** A monotone formula is a Boolean formula with no NOT signs.

If you put together what we did with $TH$ and use induction you can prove the following:

**Theorem** Let $X_1, \ldots, X_N$ each be a threshold $TH_A(t, m)$ but all using DIFFERENT players. Let $F(X_1, \ldots, X_N)$ be a monotone Boolean formula where each $X_i$ appears only once. Then Zelda can do ideal secret sharing where only sets that satisfy $F(X_1, \ldots, X_N)$ can learn the secret.

Routine proof left to the reader. Might be on a HW or the Final.
General Theorem

Definition A monotone formula is a Boolean formula with no NOT signs.

If you put together what we did with $TH$ and use induction you can prove the following:

Theorem Let $X_1, \ldots, X_N$ each be a threshold $TH_A(t, m)$ but all using DIFFERENT players.
Let $F(X_1, \ldots, X_N)$ be a monotone Boolean formula where each $X_i$ appears only once. Then Zelda can do ideal secret sharing where only sets that satisfy $F(X_1, \ldots, X_N)$ can learn the secret.

Routine proof left to the reader. Might be on a HW or the Final.
Access Structures That Admit Ideal Sec. Sharing

1. Threshold Secret sharing: if \( t \) or more get together. We did this.

2. Monotone Boolean Formulas of Threshold where every set of players appears only once. We did this.

3. Monotone Span Programs (Omitted – it's a Matrix Thing) We did not do this and will not.
Access Structures That Admit Ideal Sec. Sharing

1. Threshold Secret sharing: if $t$ or more get together. We did this.
Access Structures That Admit Ideal Sec. Sharing

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Access Structures That Admit Ideal Sec. Sharing

1. Threshold Secret sharing: if \( t \) or more get together. **We did this.**

2. Monotone Boolean Formulas of Threshold where every set of players appears only once. **We did this.**

3. Monotone Span Programs (Omitted – it’s a Matrix Thing) **We did not do this and will not.**
Access Structures That Do Not Admit Ideal Sec Sharing

1. $(A_1 \land A_2) \lor (A_2 \land A_3) \lor (A_3 \land A_4)$

2. $(A_1 \land A_2 \land A_3) \lor (A_1 \land A_4) \lor (A_2 \land A_4) \lor (A_3 \lor A_4)$ (Captain and Crew)

$A_1, A_2, A_3$ is the crew, and $A_4$ is the captain. Entire crew, or captain and 1 crew, can get shares.

3. $(A_1 \land A_2 \land A_3) \lor (A_1 \land A_4) \lor (A_2 \land A_4) \lor (A_3 \lor A_4)$ (Captain and Rival)

$A_1, A_2, A_3$ is the crew, $A_3$ is a rival, $A_4$ is the captain. Entire crew, or captain and 1 crew who is NOT rival, can get shares.

4. Any access structure that contains any of the above.

In all of the above, all get a share of size 1.

5. And this is optimal.
Access Structures That Do Not Admit Ideal Sec Sharing

1. \((A_1 \land A_2) \lor (A_2 \land A_3) \lor (A_3 \land A_4)\)
Access Structures That Do Not Admit Ideal Sec Sharing

1. \((A_1 \land A_2) \lor (A_2 \land A_3) \lor (A_3 \land A_4)\)
2. \((A_1 \land A_2 \land A_3) \lor (A_1 \land A_4) \lor (A_2 \land A_4) \lor (A_3 \lor A_4)\) (Captain and Crew)\n
\(A_1, A_2, A_3\) is the crew, and \(A_4\) is the captain. Entire crew, or captain and 1 crew, can get \(s\).
Access Structures That Do Not Admit Ideal Sec Sharing

1. \((A_1 \land A_2) \lor (A_2 \land A_3) \lor (A_3 \land A_4)\)
2. \((A_1 \land A_2 \land A_3) \lor (A_1 \land A_4) \lor (A_2 \land A_4) \lor (A_3 \lor A_4)\) (Captain and Crew) \(A_1, A_2, A_3\) is the crew, and \(A_4\) is the captain. Entire crew, or captain and 1 crew, can get \(s\).
3. \((A_1 \land A_2 \land A_3) \lor (A_1 \land A_4) \lor (A_2 \land A_4)\) (Captain and Rival) \(A_1, A_2, A_3\) is the crew, \(A_3\) is a rival, \(A_4\) is the captain. Entire crew, or captain and 1 crew who is NOT rival, can get \(s\).
Access Structures That Do Not Admit Ideal Sec Sharing

1. \((A_1 \land A_2) \lor (A_2 \land A_3) \lor (A_3 \land A_4)\)

2. \((A_1 \land A_2 \land A_3) \lor (A_1 \land A_4) \lor (A_2 \land A_4) \lor (A_3 \lor A_4)\) (Captain and Crew) \(A_1, A_2, A_3\) is the crew, and \(A_4\) is the captain. Entire crew, or captain and 1 crew, can get \(s\).

3. \((A_1 \land A_2 \land A_3) \lor (A_1 \land A_4) \lor (A_2 \land A_4)\) (Captain and Rival) \(A_1, A_2, A_3\) is the crew, \(A_3\) is a rival, \(A_4\) is the captain. Entire crew, or captain and 1 crew who is NOT rival, can get \(s\).

4. Any access structure that contains any of the above.

In all of the above, all get a share of size \(1.5n\) and this is optimal.
Zelda wants to share secret such that:

1. If $A_1, A_2, A_3$ get together they can get secret.
2. If $A_1, A_4$ get together they can get secret.
3. If $A_2, A_4$ get together they can get secret.

By the last slide we know that CANNOT do ideal secret sharing.
Can Zelda Always Secret Share?

Zelda wants to share secret such that:

1. If $A_1, A_2, A_3$ get together they can get secret.
2. If $A_1, A_4$ get together they can get secret.
3. If $A_2, A_4$ get together they can get secret.

By the last slide we know that CANNOT do ideal secret sharing. Can Zelda do secret sharing? VOTE Yes or NO.
Can Zelda Always Secret Share?

Zelda wants to share secret such that:

1. If $A_1, A_2, A_3$ get together they can get secret.
2. If $A_1, A_4$ get together they can get secret.
3. If $A_2, A_4$ get together they can get secret.

By the last slide we know that CANNOT do ideal secret sharing. Can Zelda do secret sharing? VOTE Yes or NO.
YES- but do not use polynomials, use the random string method.
Open Question

Known
Open Question

**Known**

1. Using Random String Method every Access Structure with \( m \) people has a secret sharing scheme with \( 2^m n \) sized shares.
Open Question

**Known**

1. Using Random String Method every Access Structure with $m$ people has a secret sharing scheme with $2^m n$ sized shares.
2. Threshold and many other Access Structures can do secret sharing with $n$-sized shares.
Open Question

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Open Determine for every access structure the functions $f(n)$ and $g(n)$ such that
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**Open** Determine for every access structure the functions \( f(n) \) and \( g(n) \) such that

1. (\( \exists \)) Scheme where everyone gets \( \leq f(n) \) sized share.
Open Question

**Known**

1. Using Random String Method every Access Structure with $m$ people has a secret sharing scheme with $2^m n$ sized shares.
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**Open** Determine for every access structure the functions $f(n)$ and $g(n)$ such that

1. ($\exists$) Scheme where everyone gets $\leq f(n)$ sized share.
2. ($\forall$) Scheme someone gets $\geq g(n)$ sized share.
Open Question

Known

1. Using Random String Method every Access Structure with $m$ people has a secret sharing scheme with $2^m n$ sized shares.
2. Threshold and many other Access Structures can do secret sharing with $n$-sized shares.

Open Determine for every access structure the functions $f(n)$ and $g(n)$ such that

1. $(\exists)$ Scheme where everyone gets $\leq f(n)$ sized share.
2. $(\forall)$ Scheme someone gets $\geq g(n)$ sized share.
3. $f(n)$ and $g(n)$ are close together.