Solutions to HW11 Problems
BILL, RECORD LECTURE!!!!

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FILL OUT ALL COURSE EVALS

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FILL THEM OUT! Three reasons.

1. Teachers reads them and uses it to help their teaching. Especially the comments.
2. The teaching eval comm reads them to help teachers with weak spots. I was the originator and the chair of the Teaching Eval Comm for 12 years. I was frustrated with courses with not-that-many evals filled out! (Nobody should be in any admin position for more than 5 years!)
3. These evals are used in the promotion process (e.g., Tenure). It is our hope that because the Teaching Eval Comm helps people become better teachers, there is NO bad teaching so this is not an obstacle for promotion.
4. And you can help us! By filling out the forms!
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On TV likely not in Real Life

1. Alice is interrogating Bob but actually Alice is a double agent on Bob's side.
2. Alice has to pass Bob information, telling what to say, while she is interrogating him. How does she do this?
3. ON TV: Alice punches Bob in Morse code!
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Z has \( s \). Will share with \( A_1, \ldots, A_6 \). Access Structure:

\[
\begin{align*}
\{ A_1, A_2 \}, \\
\{ A_2, A_3 \}, \\
\{ A_3, A_4 \}, \\
\{ A_4, A_5 \}, \\
\{ A_5, A_6 \}, \\
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Give Info Theoretic Sec Sharing Scheme.
Z has $s$. Will share with $A_1, \ldots, A_6$. Access Structure:
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\{A_1, A_3, A_5\}.
\]

Give Info Theoretic Sec Sharing Scheme.

State what sizes of shares everyone gets.
HW11, Problem 1 SOLUTION: The protocol

The r's below are all separate and independent.

Z gen rand \( r \in \{0, 1\}^n \).

\( A_1 : (12^r), \quad A_2 : (23^r \oplus s) \).

\( Z \) gen rand \( r \in \{0, 1\}^n \).

\( A_3 : (23^r), \quad A_4 : (23^r \oplus s) \).

\( Z \) gen rand \( r \in \{0, 1\}^n \).

\( A_5 : (34^r), \quad A_6 : (34^r \oplus s) \).

\( Z \) gen rand \( r_1, r_2 \in \{0, 1\}^n \).

\( A_1 : (135^r), \quad A_3 : (135^s), \quad A_4 : (135^s \oplus r_1 \oplus r_2) \).
HW11, Problem 1 SOLUTION: The protocol

**Note** The $r$’s below are all separate and independent.
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Z gen rand \( r \in \{0, 1\}^n \). \( A_3:(34, r), A_4:(34, r \oplus s) \).

Z gen rand \( r \in \{0, 1\}^n \). \( A_4:(45, r), A_5:(45, r \oplus s) \).
HW11, Problem 1 SOLUTION: The protocol

Note The $r$’s below are all separate and independent.

Z gen rand \( r \in \{0, 1\}^n \). \( A_1: (12, r) \), \( A_2: (12, r \oplus s) \).

Z gen rand \( r \in \{0, 1\}^n \). \( A_2: (23, r) \), \( A_3: (23, r \oplus s) \).

Z gen rand \( r \in \{0, 1\}^n \). \( A_3: (34, r) \), \( A_4: (34, r \oplus s) \).

Z gen rand \( r \in \{0, 1\}^n \). \( A_4: (45, r) \), \( A_5: (45, r \oplus s) \).

Z gen rand \( r \in \{0, 1\}^n \). \( A_5: (56, r) \), \( A_6: (56, r \oplus s) \).
**Note** The $r$’s below are all separate and independent.

Z gen rand $r \in \{0, 1\}^n$. $A_1:(12, r)$, $A_2:(12, r \oplus s)$.

Z gen rand $r \in \{0, 1\}^n$. $A_2:(23, r)$, $A_3:(23, r \oplus s)$.

Z gen rand $r \in \{0, 1\}^n$. $A_3:(34, r)$, $A_4:(34, r \oplus s)$.

Z gen rand $r \in \{0, 1\}^n$. $A_4:(45, r)$, $A_5:(45, r \oplus s)$.

Z gen rand $r \in \{0, 1\}^n$. $A_5:(56, r)$, $A_6:(56, r \oplus s)$.

Z gen rand $r_1, r_2 \in \{0, 1\}^n$. $A_1:(135, r_1)$, $A_3:(135, r_2)$, $A_4:(135, s \oplus r_1 \oplus r_2)$. 
HW11, Problem 1 SOLUTION: Size of Shares

1. $A_1$ is in 2 protocols: 12 and 135, so gets $2s + O(1)$.

2. $A_2$ is in 2 protocols: 12 and 23, so gets $2s + O(1)$.

3. $A_3$ is in 3 protocols: 23, 34, 135, so gets $3s + O(1)$.

4. $A_4$ is in 2 protocols: 34, 45, so gets $2s + O(1)$.

5. $A_5$ is in 3 protocols: 45, 56, 135, so gets $2s + O(1)$.

6. $A_6$ is in 1 protocols: 56, so gets $s + O(1)$. 
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$A_5$ is in 3 protocols: 45, 56, 135, so gets $2|s| + O(1)$.

$A_6$ is in 1 protocol: 56, so gets $|s| + O(1)$. 
Z is doing info-theoretic (3, 6) secret sharing with $A_1, A_2, A_3, A_4, A_5, A_6$. She uses polynomial method with $p = 37$. She has a “brilliant” idea: Rather than share ONE secret of $\mathbb{Z}_p$, she will share two secrets! Here is her plan.

- She wants to share $s_1, s_2 \in \mathbb{Z}_p$.
- She picks ONE random $r \in \mathbb{Z}_p$.
- She formulates the polynomial $f(x) = rx^2 + s_1x + s_2 \pmod{p}$.
- For $1 \leq i \leq 6$ she gives $A_i$ the number $f(i)$.
- If any three get together they will have three points on a degree-2 equation and hence they can find the equation $f(x)$, and hence they can find $s_1, s_2$.

Show why this is a BAD idea.
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All math is mod $p$. 
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$A_1$ has $f(1) = r + s_1 + s_2$.
$A_2$ has $f(2) = 4r + 2s_1 + s_2$. 
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$A_1$ has $f(1) = r + s_1 + s_2$.
$A_2$ has $f(2) = 4r + 2s_1 + s_2$.

Mult the first eq by 4 and then subtract:

$4f(1) - f(2) = 2s_1 + 3s_2$.

This limits the number of poss for $(s_1, s_2)$ and hence leaks info.
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$A_2$ has $f(2) = 4r + 2s_1 + s_2$.

$A_1$ and $A_2$ know $4f(1) - f(2) = 2s_1 + 3s_2$.
This LIMITS the number of poss for $(s_1, s_2)$ and hence leaks info.
$p = 37$. All math is mod 37.

$f(1) = 9$

$f(2) = 10$
HW11, Problem 2. Example of Solution

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\[ f(1) = 9 \]
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HW11, Problem 2. Example of Solution

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\( 4f(1) - f(2) = 2s_1 + 3s_2 \). Hence
\( 2s_1 + 3s_2 = 4f(1) - f(2) = 36 - 10 = 16. \)
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$2s_1 + 3s_2 = 4f(1) - f(2) = 36 - 10 = 16$.
Multiply by inverse of 2, which is 9.

$$18s_1 + 27s_2 = 9 \times 16 = 9 \times -1 = -9 = 8$$

$$s_1 + 10s_2 = 8$$

$$s_1 = 8 - 10s_2.$$
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\[ 18s_1 + 27s_2 = 9 \times 16 = 9 \times -1 = -9 = 8 \]

\[ s_1 + 10s_2 = 8 \]

\[ s_1 = 8 - 10s_2. \]

Once \( s_2 \) is known, \( s_1 \) is known. Hence there are only 37 options for \((s_1, s_2)\) instead of \(37^2\).
In class (Nov 16 lecture A-B-Love-Cards) we did several protocols (using cards and other devices) such that A and B can determine if they want a second date; however, if A wants a second date but B doesn’t B doesn’t know that (and vice versa).

A, B, C, D all get together for dinner. They want to see if they want to have dinner again. If ALL want to dine again, they will. If at least ONE person does not, they won’t.

Come up with a protocol so that at the end they all know if they want to have dinner together again, but if the answer is NO then the people who voted NO do not know how anyone else voted. You can use any of the devices in the talk on A and B.
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A, B, C, D all come with two cards— one opaque and one glass. They all put their card in a box. Glass if YES, opaque if NO.
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Light is shown through the box.
A, B, C, D all come with two cards- one opaque and one glass. They all put their card in a box. Glass if YES, opaque if NO.

Light is shown through the box.
If light goes all the way through then all said glass, so YES, they all dine together.
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Light is shown through the box.
If light goes all the way through then all said glass, so YES, they all dine together.
If light does not go through then at least one person said NO, but side from that person nobody knows who it was.