BILL, RECORD LECTURE!!!!

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Reminder

Types of Attacks
Recall Types of Attacks

**Ciphertext Only Attack (COA)**  Eve just gets to see ciphertext.
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**Known Plaintext Attack (KPA)** Eve just gets to see ciphertext and some old ciphertext-plaintext pairs.
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Brute Force Attack (BFA) Try every key.
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**Known Plaintext Attack (KPA)** Eve just gets to see ciphertext and some old ciphertext-plaintext pairs.

**Brute Force Attack (BFA)** Try every key.

For all of these attacks. Eve’s goal is to find out something about the plaintext she did not already know.
Finding out what was sent is not the only measure of success.
Learning With Errors: Private Key
Solving a System of Equations over Mod

Quick, find a solution to

\[40k_1 + 28k_2 + 111k_3 + 7k_4 \equiv 19 \pmod{191}\].

One answer is \(k_1 = 170, k_2 = 39, k_3 = 3, k_4 = 1\).

How did I know \((170, 39, 3, 1)\) worked?

Am I a math genius?

(Spoiler Alert: No)
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$$40 \times 170 + 28 \times 39 + 111 \times 3 + 7 \times 1 \equiv -40 \times 21 + 137 + 340$$

$$\equiv -840 + 137 + 149 \equiv -76 + 137 + 149 \equiv 19$$
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I Swapped Question and Answer!

On the TV show JEOPARDY the host gives the **answer** and the players give the **question**. Same here.
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For a version of the Jeopardy theme song with words see https://www.youtube.com/watch?v=A7UgxCayfV0
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Our domain is mod 191 throughout.
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3. I calculated $170 \times 40 + 39 \times 28 + 3 \times 111 + 1 \times 7 \equiv 19$.
4. I know $40k_1 + 28k_2 + 111zk_3 + 7k_4 \equiv 19 \pmod{191}$ has answer $(170, 39, 3, 1)$. 
Important Definition: DOT Product

We redo our math and introduce a notation.
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Generally:

$\left( k_1, \ldots, k_n \right) \cdot \left( r_1, \ldots, r_n \right) = k_1 \times r_1 + \cdots + k_n \times r_n$.

We will always be doing this Mod $p$. 
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$$(k_1, \ldots, k_n) \cdot (r_1, \ldots, r_n) = k_1 \times r_1 + \cdots + k_n \times r_n.$$

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Example of an Idea for a Cipher

1. Alice and Bob both have private key (170, 39, 3, 1).
2. If Alice wants to send 0 she sends Bob an equation that (170, 39, 3, 1) DOES solve. She can generate such an equation as I did above.
3. If Alice wants to send 1 she sends Bob an equation that (170, 39, 3, 1) DOES NOT solve. She can generate such an equation by doing what I did above and add 1.

▶ Would use a bigger mod and a longer equation in real life.
▶ This cipher only allows transmitting one bit.
Example of an Idea for a Cipher

1. Alice and Bob both have private key $(170, 39, 3, 1)$. Alice and Bob and Eve have public Key 191.

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Example of Using This Cipher

**Private Key** (170, 39, 3, 1). Both Alice and Bob have this.

**Public Info** 191, the mod. All math is mod 191.

**Alice Wants to Send** $b \in \{0, 1\}$.
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3. To send $b$ Alice sends $(40, 28, 111, 7; 19 + b)$. 
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4. If Bob gets \((40, 28, 111, 7; 19)\) he will do
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If Bob gets \((40, 28, 111, 7; 20)\) he will do \((40, 20, 111, 7) \cdot (170, 39, 3, 1) \equiv 19\), note \(19 \not\equiv 20\) and know \(b = 1\).
Eve Can Crack This: Eve’s View

**Private Key** \((k_1, k_2, k_3, k_4)\). Both Alice and Bob have this.

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1. Alice picks random set of 4 elements: \((40, 28, 111, 7)\).
2. Alice computes \((40, 28, 111, 7) \cdot (k_1, k_2, k_3, k_4) \equiv C\) (Eve does not see \(C\))
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KPA attack Eve later finds out that \(b = 0\), so \(C \equiv 19\). Eve knows:
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**KPA attack** Eve later finds out that \(b = 0\), so \(C \equiv 19\). Eve knows:

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40k_1 + 28k_2 + 111k_3 + 7k_4 \equiv 19
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Number of possibilities for key \((k_1, k_2, k_3, k_4)\) is now 191^3. If sees more messages can cut down search space to one possibility.
Protocol made a sharp distinction between:

- Key is solution.
- Key is not solution.
How to Fix This? Recall the Protocol

Protocol made a sharp distinction between:

- Key is solution.
- Key is not solution.

That is too sharp. Instead we will do distinction between:

- Key is close to a solution.
- Key is far from a solution.
Notation We Will Need

\( e \in^\prime A \) means that \( e \) is picked uniformly at random from the set \( A \).
Notation We Will Need

e ∈ \mathbb{R}^A \text{ means that } e \text{ is picked uniformly at random from the set } A.

We will pick our error uniformly.
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When LWE is really used they pick the error with a Gaussian around 0.
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\( e \in^r A \) means that \( e \) is picked unif at random from the set \( A \).

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When LWE is really used they pick the error with a Gaussian around 0.

We are doing it in a way that is not used but better for education.
Example of Better Cipher

**Private Key** (170, 39, 3, 1). **Public Info** mod 191.

1. Alice picks random set of 4 elements: (40, 28, 111, 7).
2. Alice computes $(40, 28, 111, 7) \cdot (170, 39, 3, 1) \equiv 19$.
3. Bit $b$: A sends $(40, 28, 111, 7; 19 + e + 50b)$. $e \in \{-1, 0, 1\}$.
4. If Bob gets $(40, 28, 111, 7; 19 + e)$ he will do $(40, 20, 111, 7) \cdot (170, 39, 3, 1) \equiv 19 \sim 19 + e$, so bit is 0. If Bob gets $(40, 28, 111, 7; 19 + e + 50)$ he will do $(40, 20, 111, 7) \cdot (170, 39, 3, 1) \equiv 19 \not\sim 19 + e + 50$ so bit is 1.

$e \in \{-1, 0, 1\}$. Note that $-1 \equiv 190$. $e \in \{-1, 0, 1\}$. In real system $e \in \{-\gamma, \ldots, \gamma\}$, $\gamma$ a param.

We picked 50 as our big number. In real system use $\sim p^4$. 
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3. Bit \(b\): A sends \((40, 28, 111, 7; 19 + e + 50b)\).
   \(e \in \{−1, 0, 1\}\).
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4. If Bob gets \((40, 28, 111, 7; 19 + e)\) he will do
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Example of Better Cipher

Private Key \((170, 39, 3, 1)\). Public Info \(\mod 191\).

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\((40, 20, 111, 7) \cdot (170, 39, 3, 1) \equiv 19 \not\sim 19 + e + 50\) so bit is 1.
Example of Better Cipher

Private Key \((170, 39, 3, 1)\). Public Info mod 191.

1. Alice picks random set of 4 elements: \((40, 28, 111, 7)\).
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3. Bit \(b\): A sends \((40, 28, 111, 7; 19 + e + 50b)\).
   \(e \in \{-1, 0, 1\}\).

4. If Bob gets \((40, 28, 111, 7; 19 + e)\) he will do \((40, 20, 111, 7) \cdot (170, 39, 3, 1) \equiv 19 \sim 19 + e\), so bit is 0.
   If Bob gets \((40, 28, 111, 7; 19 + e + 50)\) he will do \((40, 20, 111, 7) \cdot (170, 39, 3, 1) \equiv 19 \not\sim 19 + e + 50\) so bit is 1.

\(\Rightarrow e \in \{-1, 0, 1\}\). Note that \(-1 \equiv 190\).
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1. Alice picks random set of 4 elements: (40, 28, 111, 7).
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\[ e \in \{−1, 0, 1\}. \text{ Note that } −1 \equiv 190. \]

\[ e \in \{−1, 0, 1\}. \text{ In real system } e \in \{−\gamma, \ldots, \gamma\}, \gamma \text{ a param.} \]
Example of Better Cipher

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\(\triangleright\) \(e \in \{-1, 0, 1\}\). In real system \(e \in \{-\gamma, \ldots, \gamma\}\), \(\gamma\) a param.

\(\triangleright\) We picked 50 as our big number. In real system use \(\sim \frac{p}{4}\).
Floor Ceiling Convention; Vector Notation

When we write something like $\frac{p}{4}$ where $p$ is odd we really mean

$$\left\lfloor \frac{p}{4} \right\rfloor$$
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In our concrete examples we had things like

The Key is (1, 2, 3, 40)
When we write something like $\frac{p}{4}$ where $p$ is odd we really mean

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In our concrete examples we had things like

The Key is (1, 2, 3, 40)

We will now use $\vec{k}$ for the key of length $n$
When we write something like $\frac{p}{4}$ where $p$ is odd we really mean $\left\lfloor \frac{p}{4} \right\rfloor$

In our concrete examples we had things like

The Key is $(1, 2, 3, 40)$

We will now use $\vec{k}$ for the key of length $n$

We will now use $\vec{r}$ for a random vector of length $n$. 
Private Key LWE Cipher

Private Key $\vec{k}$. Both Alice and Bob have this.
Public Info $p, \gamma$. $p$ is prime. All math is mod $p$. 

Alice Wants to Send $b \in \{0, 1\}$.
1. Alice picks random vector $\vec{r}$.
2. Alice computes $\vec{r} \cdot \vec{k} \equiv C$ and $e \in \{-\gamma, \ldots, \gamma\}$.
3. To send $b$ Alice sends $(\vec{r}; D)$ where $D \equiv C + e + bp$.
4. Bob computes $\vec{r} \cdot \vec{k} \equiv C$. If $D \sim C$ then $b = 0$. If $D \sim C + p$ then $b = 1$.

Is this a good cipher? Easy to use? Secure? Discuss.
Private Key $\vec{k}$. Both Alice and Bob have this.
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Private Key LWE Cipher: Pick $\gamma$ so Works

If $b = 0$ then Bob compares $C$ to $C + e$. Diff: $e \in \{-\gamma, ..., \gamma\}$.

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Need these intervals are disjoint. Two intervals mod $p$ are disjoint iff when you shift them they are disjoint. We want to shift them to avoid wrap around. Shift both by $\gamma$. Need \{0, ..., 2$\gamma$\} and \{\frac{p}{4}, ..., 2$\gamma$ + \frac{p}{4}\} disjoint. So need $2\gamma < \frac{p}{4}$ and $2\gamma + \frac{p}{4} < p$. $\gamma < \frac{p}{16}$ suffices. (Actually $\frac{p}{8}$ suffices, but we will use $\frac{p}{16}$.)

If $b = 0$: Bob sees that diff is in \{0, ..., p/16\} = \{0, ..., p/16\} \cup \{15p/16, ..., p - 1\}.

If $b = 1$: Bob sees that diff is in \{3p/16, ..., 5p/16\}. 
Private Key LWE Cipher: Pick $\gamma$ so Works

- If $b = 0$ then Bob compares $C$ to $C + e$.
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Why did I use a Prime?

Recall the protocol:

- **Private Key** $\vec{k}$. Both Alice and Bob have this.
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What problem does Eve need to solve to find the key?
What problem does Eve need to solve to find the key?

**Learning With Errors Problem (LWE)** Eve is given $p, \gamma$ and told there is a key $\vec{k}$ that she wants to find.

Eve is given a set of tuples $(\vec{r}, D)$ and told that $\vec{r} \cdot \vec{k} - D \equiv e \in \{ -\gamma, \ldots, \gamma \}$. (Eve is not told $e$, just that $e \in \{ -\gamma, \ldots, \gamma \}$.) From these noisy equations she wants to learn $\vec{k}$.

Hard? This is thought to be a hard problem. (We will go into why LWE is thought to be hard when we do LWE-public, which won't be for a while.)
Private Key LWE Cipher: Security

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**Theorem About Security**

**Informal Theorem** If Eve can crack LWE-private cipher then Eve can solve the LWE-problem. Note that this is the direction you want.

Proof

We won't prove this, but we note that it requires some work.

1. Since LWE-problem is thought to be hard, the LWE-private cipher is thought to be hard-to-crack.
2. So why is this cipher not used? Discuss.

- Only one bit.
- Can be modified to transmit more bits.
- For private-key crypto, better schemes are known.
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Theorem About Security: Very Nice PRO

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Proof We won’t prove this, but we note that it requires some work.

1. A problem that plagues complexity theory is that a problem can have a bad worst-case but a reasonable average-case.
2. For LWE this is NOT an issue.
3. Hence the assumption that LWE is hard for worst case already gives you hard for avg case.
BILL, STOP RECORDING LECTURE!!!!

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