BILL TAPE LECTURE
Diffie-Helman Key Exchange
Summary of Where We Are

1. Finding primes $p$ such that $p - 1 = 2q$, where $q$ is a prime, EASY.
2. Given such a $p$, finding generator $g$, EASY.
3. Given such a $p$, finding generator $g \in \{p^3, \ldots, 2p^3\}$, EASY.
4. Given $p$, $g$, $a$, finding $g^a \equiv a \pmod{p}$, EASY.

The following problem thought to be hard:

Input: prime $p$, generator $g \in \{p^3, \ldots, 2p^3\}$, and $a$.

Output: The $x$ such that $g^x \equiv a \pmod{p}$.

The problem thought to be hard is essentially the discrete log problem, though we have safeguarded against easy instances.

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Convention (Possibly Repeated)

For the rest of the slides on Diffie-Hellman Key Exchange there will always be a prime $p$ that we are considering and a generator $g \in \{\frac{p}{3}, \frac{2p}{3}\}$. We omit the bounds on $g$.

**ALL** arithmetic done from that point on is mod $p$.

**ALL** numbers are in $\{1, \ldots, p - 1\}$. 
The Diffie-Hellman Key Exchange

Alice & Bob want to establish a secret $s$ w/o meeting.

1. Alice finds $(p, g)$, $p$ of length $L$, $g$ gen for $\mathbb{Z}_p^*$.
2. Alice sends $(p, g)$ to Bob (Eve can see it).
3. Alice picks rand $a$. Alice computes $g^a \pmod{p}$ and sends it to Bob (Eve can see it).
4. Bob picks rand $b$. Bob computes $g^b \pmod{p}$ and sends it to Alice (Eve can see it).
5. Alice computes $(g^b)^a = g^{ab} \pmod{p}$.
6. Bob computes $(g^a)^b = g^{ab} \pmod{p}$.
7. $s = g^{ab}$ is the shared secret.

PRO:
Alice and Bob can execute the protocol easily.

Biggest PRO:
Alice and Bob never had to meet!

Question:
Can Eve find out $s$?
The Diffie-Hellman Key Exchange

Alice & Bob want to establish a secret $s$ w/o meeting. Security parameter $L$.

1. Alice finds a $(p, g)$, $p$ of length $L$, $g$ gen for $\mathbb{Z}_p^\ast$.
2. Alice sends $(p, g)$ to Bob (Eve can see it).
3. Alice picks $\text{rand } a$. Alice computes $g^a \pmod p$ and sends it to Bob (Eve can see it).
4. Bob picks $\text{rand } b$. Bob computes $g^b \pmod p$ and sends it to Alice (Eve can see it).
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Have Students DO The DH Key Exchange

Pick out two students who I will call Alice and Bob.
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3. ALICE: Yell out $(p, g)$. 
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10. At the count of 3 both yell out your number at the same time.
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What Do We Really Know about Diffie-Hellman?

If Eve can compute Discrete Log quickly then she can crack DH:

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2. Eve computes Discrete Log to find $a$, $b$.
3. Eve computes $g^{ab} \mod p$.

Question: If Eve can crack DH then Eve can compute Discrete Log.

VOTE: Y, N, UNKNOWN TO SCIENCE.

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Definition Let $DHF$ be the following function:

**Inputs:** $p, g, g^a, g^b$ (note that $a, b$ are not the input)

**Outputs:** $g^{ab}$.

**Obvious Theorem:** If Alice can crack Diffie-Hellman quickly then Alice can compute $DHF$ quickly.
Hardness Assumption

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**Hardness assumption:** $DHF$ is hard to compute.
About the Hardness Assumption

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About the Hardness Assumption

**Hardness assumption:** $DHF$ is hard to compute.

Do we believe the hardness assumption? Yes.

1. Nobody has found a way to solve $DHF$ quickly that does not involve solving Discrete Log.
2. Discrete Log is believed to be hard.
3. Still, would be nice to have a key exchange based on DL.
How Can Alice and Bob Use DH Key Exchange?
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Alice and Bob want to set up a crypto system and NEVER MEET. Can DH help them?
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**Example** Alice wants to tell Bob to use Book Cipher with *Bounded Queries in Recursion theory by Gasarch and Martin* [https://www.amazon.com/Bounded-Queries-Recursion-Progress-Computer/dp/1461268486](https://www.amazon.com/Bounded-Queries-Recursion-Progress-Computer/dp/1461268486) a book that sold 2 copies last year AND Amazon has the author’s name as **William Levine** (I do not now why).
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Next Slide continues this discussion.
With DH Alice and Bob do not the Message

Recall:

1. Alice finds a \((p, g)\), \(p\) of length \(L\), \(g\) gen for \(\mathbb{Z}_p^*\).

2. Alice sends \((p, g)\) to Bob (Eve can see it).

3. Alice picks \(\text{rand}\) \(a\). Alice computes \(g^a\) and broadcasts it.

4. Bob picks \(\text{rand}\) \(b\). Bob computes \(g^b\) and broadcasts it.

5. Alice computes \((g^b)^a = g^{ab}\) (mod \(p\)).

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7. \(s = g^{ab}\) is the shared secret.

At the end Alice and Bob have \(s\) but \(s\) has no meaning!

\(s\) is not going to be Bounded Queries in Recursion Theory. \(s\) is going to be some random number in \(\{1, \ldots, p-1\}\).
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At the end Alice and Bob have \(s\) but \(s\) has no meaning! \(s\) is not going to be \textbf{Bounded Queries in Recursion Theory}. \(s\) is going to be some random number in \(\{1, \ldots, p-1\}\).
How can Alice and Bob Use $s$?

$s$ is random.
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**When life gives you a lemon, make lemonade.**
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*When life gives you a lemon, make lemonade.*

*When life gives you a random string, use a one-time pad.*
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1. Alice and Bob do DH and have shared string $s$.
2. Alice uses $s$ as the key for a 1-time pad to tell Bob the name of the Book for Book Cipher.
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This is not quite what people do but its the idea. Next slide is **ElGamal Public Key Crypto Systems** which is what people do.
Note really 1-Time Pad

Usual 1-Time Pad messages are bit strings. Use $\oplus$. 
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In Next Protocol messages are elements of $\mathbb{Z}_p^*$. Use Mult Mod $p$. 
ElGamal is DH Made Into an Enc System

1. Alice and Bob do Diffie Hellman.
2. Alice and Bob share secret $s = g^{ab} \pmod{p}$.
3. Alice and Bob compute $s^{-1} \pmod{p}$.
4. To send $m$, Alice sends $c = ms \pmod{p}$.
5. To decrypt, Bob computes $cs^{-1} \equiv mss^{-1} \equiv m \pmod{p}$.

We omit discussion of Hardness assumption (HW)
Misc Points about DH

Key Exchange?
Possible Futures

1. DL found to be easy, so DH is cracked.
2. DHF found to be easy, so DH is cracked.
3. Slightly better but still exp algorithms for DHF are found so Alice and Bob need to up their game, but DH still secure. (IMHO this is the most likely.)
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4. Eve could measure how much time it takes for Bob to know the string and use that to narrow down the space of strings.
Eve in the Middle Attack

(Called Man in the Middle Attack in the literature.)
What if Eve could intercept both messages and replace them.

1. Alice sends $g^a$.
2. Eve intercepts the message, picks a random $a'$, and instead sends on to Bob $g^{a'}$.
3. Eve lets Bob send $g^b$ without interference.
4. Alice thinks the shared secret string is $g^{ab}$.
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**Example:** Elliptic Curve Diffie-Hellman (actually used).
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Example: Elliptic Curve Diffie-Hellman (actually used).
Example: Braid Diffie-Hellman (not actually used).
A Serious Attack on Diffie-Helman! Or is it?

The paper *Imperfect Forward Secrecy: How Diffie-Helman Fails in Practice*

https://weakdh.org/imperfect-forward-secrecy.pdf

Claims the following:

1. 82% of all vulnerable servers use the same 512 sized group.
2. After a week of preprocessing that group, they can crack DH on that group using an advanced DL algorithm.
3. Their method can be adopted to larger groups.
4. For a 1024-sized group, they could not crack, but a nation with enough computing power could.
5. The NSA may be using this approach.

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1. DH is safe from the attacks proposed in the paper.
2. The paper still has an important message:
   2.1 DO NOT use the same group (or the same $p$, $g$) all the time since some pre-computation may make it vulnerable.
   2.2 UP your game! If $L$ is so large that you think $p$ of length $L$ is safe. USE 10 $L$.
   2.3 If you publish an academic paper about cracking DL, you should have the code and make it available. See next point.
   2.4 If you actually worry about DH being cracked then tell the crypto companies or the government first. (See the fiction book Factorman. I reviewed it: https://www.cs.umd.edu/users/gasarch/BLOGPAPERS/factorman.pdf)
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