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LECTURE

September 9, 2021
Gen Sub Cipher: How to Really Crack

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General Substitution Cipher

**Def Gen Sub Cipher** with perm $f$ on $\{0, \ldots, 25\}$.

1. Encrypt via $x \rightarrow f(x)$.
2. Decrypt via $x \rightarrow f^{-1}(x)$. 
**Terminology: 1-Gram, 2-Gram, 3-Gram**

**Notation** Let $T$ be a text.
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4. One usually talks about the freq of $n$-grams.
Example of 1-Grams

Let the text be:

*Ever notice how sometimes people use math words incorrectly?*
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The following 1-gram occurs 9 times: e.
No 1-gram occurs \( \geq 10 \) times.
Example of 2-Grams

Let the text be:

*Ever notice how sometimes people use math words incorrectly?*
Example of 2-Grams

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The following 2-grams occur 2 times: me, or.
Example of 2-Grams

Let the text be:

**Ever notice how sometimes people use math words incorrectly?**

The following 2-grams occur 2 times: me, or.
The following 2-grams occur 1 time: ev, ve, er, rn, no, ot, ti, ic, eh, ho, ow, ws, so, et, ti, im, es, sp, pe, eo, op, pl, le, eu, us, se, em, ma, at, th, hw, wo, ds, in, nc, co, rr, re, ec, ct, tl, ly.
Example of 2-Grams

Let the text be:

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The following 2-grams occur 2 times: me, or.

The following 2-grams occur 1 time: ev, ve, er, rn, no, ot, ti, ic, eh, ho, ow, ws, so, et, ti, im, es, sp, pe, eo, op, pl, le, eu, us, se, em, ma, at, th, hw, wo, ds, in, nc, co, rr, re, ec, ct, tl, ly.

No 2-gram occurs $\geq 3$ times.
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Notation and Parameter for a Family of Algorithms

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1. $f_{E,n}$ is freq of $n$-grams in English. It is a $26^n$ long vector.
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3. $f_{\sigma(T),n}$ is the $26^n$-long vector of freq’s of $n$-grams in $\sigma(T)$. 

$I$ and $R$ will be parameters we discuss later. $I$ stands for Iterations and will be large (like 2000). $R$ stands for Redos and will be small (like 5).
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   - I stands for **Iterations** and will be large (like 2000).
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Stats for 1-Gram, 2-Gram, 3-Gram, 4-Gram

1. 1-grams: $f_{E,1} \cdot f_{E,1} \approx 0.065$
2. 2-grams: $f_{E,2} \cdot f_{E,2} \approx 0.0067$
3. 3-grams: $f_{E,3} \cdot f_{E,3} \approx 0.0011$
4. 4-grams: $f_{E,4} \cdot f_{E,4} \approx 0.00023$
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Contrast Shift to Gen Sub

To crack shift went through all 26 shifts $\sigma$:

1. If $f_{\sigma}(T)$ is large then $\sigma$ is correct shift. Large $\sim 0.065$.
2. If $f_{\sigma}(T)$ is small then $\sigma$ is incorrect shift. Small.
3. Important: Will always be large or small. So we have a gap.

Let's try this with gen sub, ignoring the issue of 26! perms.

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What to do?
What to do if there is no Gap?

1. Use $n$-grams instead of 1-grams.

2. If $\sigma$ is a perm and $n \in \mathbb{N}$ then good $\sigma, n = f \in E_n \cdot f \sigma(T)$.

3. Rather than view the Is-English program as a YES-NO, view it as comparative: $T_1$ looks more like English than $T_2$. 
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\textit{n-Gram Algorithm}

Input $T$. Find Freq of 1-grams and $n$-grams.
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For $r = 1$ to $R$ ($R$ is small, about 5)
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$$\sigma_r \leftarrow \sigma_{\text{init}}$$
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For $r = 1$ to $R$ ($R$ is small, about 5)

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For $i = 1$ to $I$ ($I$ is large, about 2000)
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\begin{align*}
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\text{Pick } j, k \in \{0, \ldots, 25\} \text{ at Random.}
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Pick $j, k \in \{0, \ldots, 25\}$ at Random.

Let $\sigma'$ be $\sigma_r$ with $j, k$ swapped
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Candidates for $\sigma$ are $\sigma_1, \ldots, \sigma_R$
Pick the $\sigma_r$ with max $\text{good}_{\sigma_r,n}$ or have human look at all $\sigma_r(T)$
An old question:

What came first, the chicken or the egg?
Finding Parameters: A Chicken-and-Egg Problem

An old question:
**What came first, the chicken or the egg?**

**Our Problem** We need parameters I and R so the answer looks like English. But we then need a notion of Is English that does not use a gap. Need a program to tell us that it looks like English.
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We find the parameters for texts where we know the answers.
Finding the Parameters

Do the following a large number of times:

1. Take a text $T$ of $\sim 10,000$ characters.
2. Take a random perm $\sigma$.
3. Compute $\sigma(T)$.
   (Note- We know $\sigma$ and $T$)
4. Run the $n$-gram algorithm but with no bound on the number of iterations. Stop when either
   4.1 Get original text $T$, or
   4.2 Swaps do not improve how close to English (could be in local max). In this case try again.
5. Keep track of how how many iterations suffice and how many redos suffice.
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David Zhen Found the Parameters

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For each text he generated many random perm and ran the algorithm.
Parameters for $n$-Grams

1-grams: Nothing worked.

2-grams: Nothing worked.

3-grams: I = 2000, R = 4 worked. Took ≤ 2 minutes to crack.

4-grams: I = 2000, R = 8, Took around 6 minutes to crack.

So the winner is 3-grams, with I = 2000 and R = 4.

Can we do better than 2 minutes? Can we do something clever?
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For $r = 1$ to $R$ ($R$ is small, about 5)
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        Pick $j, k \in \{0, \ldots, 25\}$ at Random Cleverly!

Let $\sigma'_{\text{init}}$ be $\sigma_r$ with $j, k$ swapped
If $f_{\sigma'}(T), n \cdot f_{\text{E}, n} > f_{\sigma_r}(T), n \cdot f_{\text{E}, n}$ then
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Candidates for $\sigma$ are $\sigma_1, \ldots, \sigma_R$ Pick the $\sigma_r$ with max good $n$ $\sigma_r$ or have human look at all $\sigma_r(T)$

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Lets say this takes less iterations. But we spend more time finding the clever swap. Is it worth it? Only way to find out is to DO IT.

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