The Vigenère Cipher

October 25, 2021
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Easy to remember and transmit.

**Example** using \( \text{dog} \).
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**Example** using $dog$.
Shift 1st letter by 3

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Jacob Prinz is a Physics Major encrypts to MOIRP VUWTC WYDDN BGOFG SDXUU
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Example using \textit{dog}.
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**Example** using *dog*.
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- Shift 2nd letter by 14
- Shift 3rd letter by 6
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Shift 3rd letter by 6
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Shift 6th letter by 6, etc.

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**Key:** \( k = (k_1, k_2, \ldots, k_n) \).

**Encrypt** (all arithmetic is mod 26)

\[
\text{Enc}(m_1, m_2, \ldots, m_N) =
\]

\[
m_1 + k_1, m_2 + k_2, \ldots, m_n + k_n,
\]

\[
m_{n+1} + k_1, m_{n+2} + k_2, \ldots, m_{n+n} + k_n,
\]

\[
\vdots
\]

**Decrypt** Decryption just reverses the process
The following three slides give three kinds of Vig Ciphers. It is a rough way to divide up types of Vig ciphers. There will be some that are not quite in any category.
The key is a sentence or paragraph in English. Memorable and not too long. For example, the following could be the key:
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When the TV game show Jeopardy had the topic CHEMISTRY they had the questions read by, not a famous chemist, but by Bryan Cranston who played a chemist on Breaking Bad. Why? Because there are no famous living chemists. This is sad!
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We will be studying this type of Vig cipher today.
The key is an entire book that Alice and Bob both have. Has to be the same edition!
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The key that Alice tells Bob can still be short since books have title and authors and edition numbers that identify them.
VIG TWO: The Vig-Book Cipher

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Alice can say to Bob:
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Alice can say to Bob:

**A Student’s Guide to Coding and Information theory by Moser and Chen, 2nd edition.**
The key is an entire book that Alice and Bob both have. Has to be the same edition!

The key that Alice tells Bob can still be short since books have title and authors and edition numbers that identify them.

Alice can say to Bob:


This is called *The Vig-Book Cipher*. We will touch on it briefly later.
The key is a very long random string of letters. Note that the key is completely random, so not memorable at all. Alice would give Bob that very long string, which is awkward.
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It is usually done with alphabet \{0, 1\} or \{0, \ldots, 9\}, not \{a, \ldots, z\}. 
Crypto Dilemma and what Amateur’s Have Done

Vig ONE: easy to use, but as we will see, Easy to Break.

One-time-Pad: hard to use, but as we will see, Hard to Break.

This is the Cryptographers Dilemma. How to make a system that is easy for Alice and Bob to use but hard for Eve to break.

In an earlier era many amateurs came up with cryptosystems that they thought were unbreakable. Their fallacies:

1. Their systems were impossible to use.
2. Their systems were only hard to break on short ciphers.
3. They assumed that the only way to break it was similar to how it was created. Example: There are 26! possible general sub ciphers, so unbreakable. NOT!
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Our Study of VIG ONE

- Size of key space?

- If keys are $\leq 20$-char then key space size $\sim 2^{60}$.

- If key can be anything then brute-force search is infeasible.

- If key is an English Sentence, Brute-force might be feasible.

- If Eve knows that Alice and Bob are fans of American History and suspects they use phrases about it (e.g., names of Vice Presidents) brute-force is even more feasible.

- Is the Vigenère cipher secure?

- Believed secure for many years.

- Might not have even been secure then.

- History of Cryptography is hard since, unlike most science, people can discover things and NOT brag about it.
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Cracking Vig cipher: Step One-find Key length

Assume $T$ is a text encoded by Vig, key length $L$ unknown.
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Assume $T$ is a text encoded by Vig, key length $L$ unknown. For $0 \leq i \leq L - 1$, letters in pos $\equiv i \pmod{L}$ – same shift. Look for a sequence of (say) 3-letters to appear (say) 4 times.
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**Example:** aiq appears in the
57-58-59th slot 87-88-89th slot 102-103-104th slot 162-163-164th slot
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**Important**: Very likely that aiq encrypted the same 3-letter sequence and hence the length of the key is a divisor of
87-57=30 102-87=15 162-102=60
The only possible $L$’s are 1,3,5,15.
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The only possible $L$’s are 1,3,5,15.

**Good Enough:** We got the key length down to a small finite set.
Important Point About Letter Freq (I)

In an English text $T$ of length $N$, where $N$ is large:

e occurs $\sim 13\%$  \hspace{1cm} t occurs $\sim 9\%$  \hspace{1cm} a occurs $\sim 8\%$

Etc- other letters have frequencies that are true for all texts.
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Etc- other letters have frequencies that are true for all texts.

In an English text $T$ of length $N$, where $N$ is large and $i \ll N$, then if you take every $i$th letter of $T$:

- $e$ occurs $\sim 13\%$
- $t$ occurs $\sim 9\%$
- $a$ occurs $\sim 8\%$

Etc- have the other letters same frequencies as normal texts.
Important Point About Letter Freq (II)

If $T$ is a text of normal English then $f_T \cdot f_E \sim 0.065$. 
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Hence, if $T$ is a text of normal English then $f_T \cdot f_T \sim 0.065$. 
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**Very Important** If $T$ is a **shifted text of normal English** then $f_T \cdot f_T \sim f_E \cdot f_E \sim 0.065$. 
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Hence, if \( T \) is a text of normal English then \( f_T \cdot f_T \sim 0.065 \).

**Very Important** If \( T \) is a **shifted text of normal English** then \( f_T \cdot f_T \sim f_E \cdot f_E \sim 0.065 \).

**Contrast** If \( T \) is not normal English or a shift of normal English then \( f_T \cdot f_T \) is probably low.
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If $T$ is a text of normal English then $f_T \cdot f_E \sim 0.065$.

Hence, if $T$ is a text of normal English then $f_T \cdot f_T \sim 0.065$.

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**Contrast** If $T$ is not normal English or a shift of normal English then $f_T \cdot f_T$ is probably low.

**Sum Up Point**

▶ If $T$ is English or shifted-English then $f_T \cdot f_T \sim 0.065$ large.
▶ If $T$ is not English nor shifted-English then $f_T \cdot f_T$ is prob small.
We Can Use This

We think that Key Length is (say) 10
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How to test this? Discuss
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1. Let $T_0$ be the letters in $T$ in spots 0, 10, 20, 30, ....
We Can Use This

We think that Key Length is (say) 10

How to test this? Discuss

1. Let $T_0$ be the letters in $T$ in spots 0, 10, 20, 30, . . . .

2. If $f_{T_0} \cdot f_{T_0}$ is large then key length probably IS 10 (or a divisor of 10).
We think that Key Length is (say) 10

How to test this? Discuss

1. Let $T_0$ be the letters in $T$ in spots 0, 10, 20, 30, . . . .
2. If $f_{T_0} \cdot f_{T_0}$ is large then key length probably IS 10 (or a divisor of 10).
3. If $f_{T_0} \cdot f_{T_0}$ is small then key length probably not 10.

Next slide generalizes this.
Our question  $T$ is ciphertext coded with Vig Cipher. Eve thinks the key length is $L$. Let $S$ be every $L$th letter of $T$. SO

$$S = T(1) T(L + 1) T(2L + 1) \cdots T(NL + 1)$$
Testing if Key Length is $L$

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- If keylength is $L$ then $S$ is a shift of every $L$th character from some English Text. Hence $f_S \cdot f_S \sim 0.065$. 
Testing if Key Length is $L$

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- If keylength is $L$ then $S$ is a shift of every $L$th character from some English Text. Hence $f_S \cdot f_S \sim 0.065$.
- If keylength is not $L$ then $S$ is a . . . a real mess!! $f_S \cdot f_S$ will be small.

**Upshot** We have a test whether some text is from the shift-cipher or not. We will use it on the every-$L$th-letter text of $T$. 
Let $K$ be the set of possible key lengths. $K$ is small. For every $L \in K$: 

- Form a stream $S$ of every $L$th character.
- Find the frequencies of that stream, $f_S$.
- Compute $Q = f_S \cdot f_S$.
- If $Q \approx 0.065$ then YES $L$ is key length.
- If $Q$ much less than 0.065 then NO $L$ is not key length.

One of these two will happen: just to make sure, check another stream.
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Another Way To Find Keylength

We presented Method ONE:

1. Find phrase of length $x$ appearing $y$ times. Differences $D$.
2. $K$ is set of divisors of all $L \in D$. Correct keylength in $K$.
3. Test $L \in K$ for key length until find one that works.

Or could try all key lengths up to a certain length, Method TWO:

1. Let $K = \{1, \ldots, 100\}$ (I am assuming key length $\leq 100$).
2. Test $L \in K$ for key length until find one that works.

Note With modern computers use Method TWO. In the pre-computation era Method ONE was used.

Question Computers reduce the need for cleverness. Is this good or bad?
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3. Test $L \in K$ for key length until find one that works.

Or could try all key lengths up to a certain length,

Method TWO:

1. Let $K = \{1, \ldots, 100\}$ (I am assuming key length $\leq 100$).
Another Way To Find Keylength

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**Note** With modern computers use Method TWO. In the pre-computation era Method ONE was used.
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Question Computers reduce the need for cleverness. Is this good or bad?
Cracking the Vig cipher: Step Two-Freq Anal

After Step One we have the key length $L$. Note:

1. Separate text $T$ into $L$ streams depending on position mod $L$.
2. For each steam try every shift and use Is English to determine which shift is correct.
3. You now know all shifts for all positions. Decrypt!
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- Every $L^{th}$ character is encrypted using the same shift.

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Using Plaintext Letter Frequencies

![Graph showing letter frequencies](image-url)
Vig-Book Cipher

October 25, 2021
A student said:

*Let’s use Vig cipher with a book for the key*

Is it a good idea? **Discuss**
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*Let’s use Vig cipher with a book for the key*

Is it a good idea? **Discuss**

1. Before modern computer era: YES.
2. Now. NO.
How to Crack the Vig-Book Cipher

**Key:** Both Key and Text have the English Lang Frequencies.

[Bar chart showing English language letter frequencies]
How to Crack the Vig-Book Cipher

Eve sees a $d$. (Recall that $d = 3$.) What does Eve know? Discuss
How to Crack the Vig-Book Cipher

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Eve knows that $(\text{First Letter in Key}) + (\text{First Letter in Text}) = 3$. Hence the following are the only possibilities for $(\text{Letter in Key, Letter in Text})$ are:

$(a, d), (z, e), (y, f), (w, g), \ldots, (b, c)$

Only 26 possibilities. What of it? **Discuss**
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Some of the pairs are more likely than others.
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Some of the pairs are more likely than others.

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3. $(a, d)$: Hmm, seems more likely than $(z, e)$.
4. Can rank which are more likely (e.g., add or mult the freqs).
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Vig-Book Cipher was Really Used

1. Benedict Arnold used the Vig-Book Cipher with the book *Commentaries on the laws of England*.

2. In WW I, Germany and a group in India that wanted independence from England, communicated using the Vig-Book Cipher. They used the book *Germany and the Germans*.

Were these good choices? NO. They are books Eve might guess.
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Each chapter of this book makes a point like those above and then illustrates the point by doing some real mathematics.

This book gives readers valuable information about how mathematics and theoretical computer science work, while teaching them some actual mathematics and computer science through examples and exercises. Much of the mathematics could be understood by a bright high school student. The points made can be understood by anyone with an interest in math, from the bright high school student to a Field’s medal winner.
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Would make a Good Ugrad Project

Cracking the book cipher would make a good ugrad project.
The Vig-Book Cipher IS Vig Cipher with Key longer than message.

1. **Weakness:** Key is English Phrase, so has freq patterns.
2. How can we strengthen?
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3. Make Key Truly Random. This is the one-time pad which we study later.