# BILL START THE RECORDING

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- 4. And you can help us! By filling out the forms!

# Threshold Secret Sharing: Length of Shares

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**Answer NO** 

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Assume that  $A_5$  gets a share of length 6. We show that the scheme is NOT info-theoretic secure.

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That is INFORMATION!!!!

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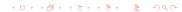
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#### You've Been Punked!

 $A_1, A_2$  CAN find s but  $A_1, A_2, A_3$  CANNOT. Thats Stupid!

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 ${\cal X}$  is closed under superset:

If  $Y \in \mathcal{X}$  and  $Y \subseteq Z$  then  $Z \in \mathcal{X}$ .

**Def** An **Access Structure** is a set of subset of  $\{A_1, \ldots, A_m\}$  closed under superset.

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**Def** A secret sharing scheme is **ideal** if all shares come from the same domain as the secret.

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To generalize this we need a better notation.

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**Notation**  $TH_A(t_1, m_1) \vee TH_B(t_2, m_2)$  means that:

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- 2.  $\geq t_2 B_1, \ldots, B_{m_2}$  can learn the secret.

Let  $TH_A(t, m)$  be the Boolean Formula that represents at least t out of m of the  $A_i$ 's.

**Example**  $TH_A(2,4)$  is

At least 2 of  $A_1, A_2, A_3, A_4$ .

**Example**  $TH_B(3,6)$  is

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- 2.  $\geq t_2 B_1, \ldots, B_{m_2}$  can learn the secret.
- 3. No other group can learn the secret (e.g.,  $A_1, A_2, B_1$  cannot)

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There is Ideal Secret Sharing for  $TH_A(t_1, m_1) \lor \cdots \lor TH_Z(t_{26}, m_{26})$ 

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- 3. Zelda and the  $Z_1, \ldots, Z_{m_{26}}$  do  $(t_{26}, m_{26})$  secret sharing.

**Note** We now have a large set of non-threshold scenarios that have ideal secret sharing.

We want that if  $\geq 2$  of  $A_1, A_2, A_3, A_4$  AND  $\geq 4$  of  $B_1, \ldots, B_7$  get together than they can learn the secret, but no other groups can. Think about it.

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$$r \oplus (r \oplus s) = s$$



 $TH_A(t_1, m_1) \wedge \cdots \wedge TH_Z(t_{26}, m_{26})$  can do secret sharing.

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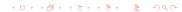
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- 6. Zelda does  $(t_{26}, m_{26})$  secret sharing of  $r_1 \oplus \cdots \oplus r_{25} \oplus s$  with  $Z_i$ 's.
- 7. If  $\geq t_1$  of  $A_i$ 's get together they can find  $r_1$ . If  $\geq t_2$  of  $B_i$ 's get together they can find  $r_2$ .  $\cdots$  If  $\geq t_{25}$  of  $Y_i$ 's get together they can find  $r_{25}$ . If  $\geq t_{26}$  of  $Z_i$ 's get together they can find  $r_1 \oplus \cdots \oplus r_{25} \oplus s$ . So if they call get together they can find

$$r_1 \oplus \cdots \oplus r_{25} \oplus (r_1 \oplus \cdots \oplus r_{25} \oplus s) = s$$



#### **General Theorem**

**Definition** A **monotone formula** is a Boolean formula with no NOT signs.

If you put together what we did with *TH* and use induction you can prove the following:

**Theorem** Let  $X_1, \ldots, X_N$  each be a threshold  $TH_A(t, m)$  but all using DIFFERENT players.

Let  $F(X_1,...,X_N)$  be a monotone Boolean formula where each  $X_i$  appears only once. Then Zelda can do ideal secret sharing where only sets that satisfy  $F(X_1,...,X_N)$  can learn the secret.

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Routine proof left to the reader. Might be on a HW or the Final.

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- 4. Any access structure that contains any of the above.

In all of the above, all get a share of size 1.5n and this is optimal.

The proof of this is difficult and hence omitted.

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Zelda wants to share secret such that:

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YES- but do not use polynomials, use the random string method.

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- 3. f(n) and g(n) are close together.