CMSC 652 HW 7-WRITTEN Due March 25

(You have LOTS of time to do this because of spring break. If you hand it in late it better be the case that your cat needed a CAT-scan.)

1. (50 points) Let $a, b \in N$. Let S be such that the number of strings of length $\le n$ is $\le n^a$. Assume that $LSAT \le S$ and the reduction runs in time $O(n^b)$. Find a number c (which depends on a, b) such that SAT is in $DTIME(O(n^c))$. (You can do this EITHER by going through the intervals-proof of $SAT \le S \rightarrow P = NP$ carefully, OR by going through the orderings-proof of $SAT \le S \rightarrow P = NP$ carefully. Either one is fine but you should know how to do both.)

2. (0 points but think about- it may be on the next HW or the midterm) Let $MOD2SAT$ be the set of all formulas that have an EVEN number of satisfying assignments. ($MODiSAT$ is the set of all formulas that have the number of sat assignments is $\equiv 0 \bmod i$.) Show that if $MOD2SAT \le_m S$ where S is sparse then $MOD2SAT \in P$. Show that if $MODiSAT \le_m S$ where S is sparse then $MODiSAT \in P$.

3. (0 points but think about it) Show that for all k, if $SAT \le_{k-tt} S$ where S is sparse then $P = NP$.

4. (0 points but you MUST hand it in) Who in the class, other than yourself, is most likely to solve P vs NP? Why?

CMSC 652 HW 7-ORAL Due March 27

(50 points) Prove that $NSPACE(\log n)$ is closed under complementation. The proof must be from first principles (e.g., you can't say 'this follows from Immerman-Szelepcsenyi Theorem.') HINT- this is a special case of Immerman-Szelepcsenyi Theorem. You should look up Immerman-Szelepcsenyi Theorem, read it, and understand it completely. The paper is *Nondeterministic Space is Closed under Complementation* by Immerman. It appeared in SIAM Journal of Computation in 1988, Volume 17. Szelepcsenyi's paper is harder to find and not as well written. There may also be notes on it on the web someplace. It may also be in complexity theory books.