The Infinite 2-ary Can Ramsey Thm

William Gasarch-U of MD
Hungarian Math Comp Problem

From the 1950 “Küuschák/Eötvös Math Competition”:

There are 1950 cans of paint. Find an $x$ such that (1) there are either $x$ cans of paint all the same color, or $x$ cans of paint that are all different colors and (2) it is possible to have neither $x + 1$ cans that are all the same nor $x + 1$ cans that are all different.

Prove with your neighbor.
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Answer is $x = 45$:
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Answer is \( x = 45 \):

1) If there are 45 different paint colors DONE

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Answer is $x = 45$:
1) If there are 45 different paint colors DONE
2) If there are 45 of the same color then DONE
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Prove with your neighbor.

Answer is \( x = 45 \):
1) If there are 45 different paint colors DONE
2) If there are 45 of the same color then DONE
3) If there are \( \leq 44 \) diff colors and each color appears \( \leq 44 \) times then \( \leq 44 \times 44 = 1936 < 1950 \) cans.
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Answer is $x = 45$:
1) If there are 45 different paint colors DONE
2) If there are 45 of the same color then DONE
3) If there are $\leq 44$ diff colors and each color appears $\leq 44$ times then $\leq 44 \times 44 = 1936 < 1950$ cans.
4) CAN have NEITHER 46 the same NOR 46 different: Color 1st 45 1, 2nd 45 2, . . . , 43nd 45 43. You’ve colored $43 \times 45 = 1935$. Color the rest 44. Have used 44 colors.
The Can Ramsey Thm is for any number of colors.

It is named “Can Ramsey” in honor of the paint can problem on the 1950 Kürschák/Eötvös Math Competition.
Thm: For every $COL : \mathbb{N} \rightarrow [c]$ there is an infinite homog set.

What if the number of colors was infinite?

Do not necessarily get a homog set since could color EVERY vertex differently. But then get infinite rainbow set.
One-Dim Can Ramsey Thm

**Thm:** Let $V$ be a countable set. Let $COL : V \rightarrow \omega$. Then there exists either an infinite homog set (all the same color) or an infinite rainb set (all diff colors).
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**Prove with your neighbor.**
Thm:  For every \( \text{COL} : \binom{\mathbb{N}}{2} \rightarrow [c] \) there is an infinite homog set.

What if the number of colors was infinite?

Do not necessarily get a homog set since could color EVERY edge differently. But then get infinite rainbow set.
Conjecture For every $COL : \binom{\mathbb{N}}{2} \to \omega$ there is an infinite homog set OR an infinite rainb set.

VOTE: TRUE, FALSE, or UNKNOWN TO SCIENCE.
**Conjecture** For every $COL : \binom{\mathbb{N}}{2} \to \omega$ there is an infinite homog set OR an infinite rainb set.

**VOTE:** TRUE, FALSE, or UNKNOWN TO SCIENCE.

FALSE:

- $COL(i, j) = \min\{i, j\}$.
- $COL(i, j) = \max\{i, j\}$. 
**Min-Homog, Max-Homog, Rainbow**

**Def:** Let $COL : \binom{\mathbb{N}}{2} \rightarrow \omega$. Let $V \subseteq \mathbb{N}$. Assume $a < b$ and $c < d$.

- $V$ is **homog** if $COL(a, b) = COL(c, d)$ iff $TRUE$.
- $V$ is **min-homog** if $COL(a, b) = COL(c, d)$ iff $a = c$.
- $V$ is **max-homog** if $COL(a, b) = COL(c, d)$ iff $b = d$.
- $V$ is **rainb** if $COL(a, b) = COL(c, d)$ iff $a = c$ and $b = d$.

**Can Ramsey Thm for $\binom{\mathbb{N}}{2}$:** For all $COL : \binom{\mathbb{N}}{2} \rightarrow \omega$, there exists an infinite set $V$ such that either $V$ is homog, min-homog, max-homog, or rainb.
Our First “Application”

We will do the following

1. Use the 4-ary Ramsey Theorem to prove the 2-ary Can Ramsey Theorem.
2. Use the 3-ary Ramsey Theorem to prove the 2-ary Can Ramsey Theorem.
3. Use a similar technique from 2-ary Ramsey Theorem to prove 2-ary Can Ramsey.
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We will do the following

1. Use the 4-ary Ramsey Theorem to prove the 2-ary Can Ramsey Theorem.

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Proof of Can Ramsey Thm for $\binom{\mathbb{N}}{2}$

We are given $COL : \binom{\mathbb{N}}{2} \rightarrow \omega$.
Want to find infinite homog OR min-homog OR max-homog OR rainbow set.

We use $COL$ to define $COL' : \binom{\mathbb{N}}{4} \rightarrow [16]$.
We then apply 4-ary Ramsey Theorem. (an “Application!”)

In the slides below $x_1 < x_2 < x_3 < x_4$.
All cases assume negation of prior cases.

**Homog** always means infinite Homog.
Pairs that begin the same way are same color

1. \( \text{COL}(x_1, x_2) = \text{COL}(x_1, x_3) \rightarrow \text{COL}'(x_1 < x_2 < x_3 < x_4) = 1. \)
2. \( \text{COL}(x_1, x_2) = \text{COL}(x_1, x_4) \rightarrow \text{COL}'(x_1 < x_2 < x_3 < x_4) = 2. \)
3. \( \text{COL}(x_1, x_3) = \text{COL}(x_1, x_4) \rightarrow \text{COL}'(x_1 < x_2 < x_3 < x_4) = 3. \)
4. \( \text{COL}(x_2, x_3) = \text{COL}(x_2, x_4) \rightarrow \text{COL}'(x_1 < x_2 < x_3 < x_4) = 4. \)

\(\text{H}\) is homog set, color 1 (rest similar)

\(\text{COL}'' : H \rightarrow \omega \) is \(\text{COL}''(x) = \text{color of all } (x, y) \text{ with } x < y \in H. \)

Use 1-dim Can Ramsey!:

**Case 1:** \(\text{COL}''\) has homog set \(H'\) then \(H'\) homog for \(\text{COL}.\)

**Case 2:** \(\text{COL}''\) has rainb set \(H'\) then \(H'\) min-homog for \(\text{COL}.\)
Pairs that End the same way are same color

1. \( \text{COL}(x_1, x_3) = \text{COL}(x_2, x_3) \rightarrow \text{COL}'(x_1 < x_2 < x_3 < x_4) = 5. \)
2. \( \text{COL}(x_1, x_4) = \text{COL}(x_2, x_4) \rightarrow \text{COL}'(x_1 < x_2 < x_3 < x_4) = 6. \)
3. \( \text{COL}(x_1, x_4) = \text{COL}(x_3, x_4) \rightarrow \text{COL}'(x_1 < x_2 < x_3 < x_4) = 7. \)
4. \( \text{COL}(x_2, x_4) = \text{COL}(x_3, x_4) \rightarrow \text{COL}'(x_1 < x_2 < x_3 < x_4) = 8. \)

\( H \) is homog set, color 5 (rest similar)
\( \text{COL}'' : H \rightarrow \omega \) is \( \text{COL}''(y) = \) color of all \((x, y)\) with \(x < y \in H\).

Use **1-dim Can Ramsey!**:

**Case 1:** \( \text{COL}'' \) has homog set \( H' \) then \( H' \) homog for \( \text{COL} \).

**Case 2:** \( \text{COL}'' \) has rainb set \( H' \) then \( H' \) max-homog for \( \text{COL} \).
Easy Homog Cases

1. \( \text{COL}(x_1, x_2) = \text{COL}(x_2, x_3) \Rightarrow \text{COL}'(x_1, x_2, x_3, x_4) = 9. \)
2. \( \text{COL}(x_1, x_2) = \text{COL}(x_2, x_4) \Rightarrow \text{COL}'(x_1, x_2, x_3, x_4) = 10. \)
3. \( \text{COL}(x_1, x_2) = \text{COL}(x_3, x_4) \Rightarrow \text{COL}'(x_1, x_2, x_3, x_4) = 11. \)
4. \( \text{COL}(x_1, x_3) = \text{COL}(x_2, x_4) \Rightarrow \text{COL}'(x_1, x_2, x_3, x_4) = 12. \)
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6. \( \text{COL}(x_2, x_3) = \text{COL}(x_1, x_4) \Rightarrow \text{COL}'(x_1, x_2, x_3, x_4) = 14. \)
7. \( \text{COL}(x_2, x_3) = \text{COL}(x_3, x_4) \Rightarrow \text{COL}'(x_1, x_2, x_3, x_4) = 15. \)

\( H \) is homog set, color 9 (rest similar)
For all \( w < x < y < z \in H \).

\[
\text{COL}(w, x) = \text{COL}(x, y) = \text{COL}(y, z).
\]

Other cases, like \( \text{COL}(w, y) = \text{COL}(x, z) \), are similar
If **NONE** of the above cases hold then \( COL'(x_1, x_2, x_3, x_4) = 16 \).
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Let $H$ be the homogenous set of $\text{COL}'$ of color 16.
That Last Case

If **NONE** of the above cases hold then $COL'(x_1, x_2, x_3, x_4) = 16$.

Let $H$ be the homogenous set of $COL'$ of color 16.

Then $H$ is a rainbow set for $COL$. Leave this to the reader, thought it is obvious.
PROS and CONS of Proof

Give me a PRO and a CON of the proof.
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**PRO:** Each Case easy. Note that Rainbow case was easy.

**CON:** Lots of Cases. Use of 4-ary hypergraph Ramsey makes finite version have large bounds.

Let $CR_2(k) =$ least $n$ s.t. $\forall \text{COL}: \binom{[n]}{2} \rightarrow \omega$, $\exists H$ of size $k$ such that either $H$ is homog, min-homog, max-homog, or rainb. If finitized, this proof obtains

$$CR_2(k) \leq R_4(k, 16) \leq 16^{16^{16^{O(k)}}}$$
PROS and CONS of Proof

Give me a PRO and a CON of the proof.

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We will give another proof which only uses 3-ary hypergraph Ramsey.
**Def** Let $COL : \binom{\mathbb{N}}{2} \rightarrow \omega$. If $c$ is a color and $v \in \mathbb{N}$ then $\deg_c(v)$ is the number of $c$-colored edges with $v$ as an endpoint.

**Note:** $\deg_c(v)$ could be infinite.
Lemma Let $X$ be infinite. Let $COL : \binom{X}{2} \to \omega$. If for every $x \in X$ and $c \in \omega$, $\deg_c(x) \leq 1$ then there is an infinite rainb set.

Prove with your Neighbor
Proof

Let $M$ be a MAXIMAL rainbow set of $X$. 

$$(\forall y \in X - M)[M \cup \{y\} \text{ is not a rainbow set}].$$

We prove $M$ is infinite.
Proof that $M$ is infinite

Assume, BWOC, that $M$ is finite. So $X - M$ is infinite.
Proof that $M$ is infinite

Assume, BWOC, that $M$ is finite. So $X - M$ is infinite.
Let $y \in X - M$. Why is $y \notin M$?
Proof that $M$ is infinite

Assume, BWOC, that $M$ is finite. So $X - M$ is infinite.

Let $y \in X - M$. Why is $y \notin M$?

Must be that:

$$(\exists u \in M, \exists \{a, b\} \in \binom{M}{2})[COL(y, u) = COL(a, b)].$$
Proof that \( M \) is infinite

Assume, BWOC, that \( M \) is finite. So \( X - M \) is infinite.

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**Informally** Map \( y \in X - M \) to the reason \( y \notin M \).
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**Informally** Map $y \in X - M$ to the reason $y \notin M$.

**Formally** If $y \in X - M$ map it to the $\{u, \{a, b\}\}$ noted above.
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Map is injective: if $y_1$ and $y_2$ both map to $(u, \{a, b\})$
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then $COL(y_1, u) = COL(y_2, u)$. 

Can't happen! $deg_c(u) \leq 1$.

So have injection from infinite $X - M$ to finite $M \times (M^2)$.

Contradiction

So $M$ is infinite.
Proof that $M$ is infinite

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then $COL(y_1, u) = COL(y_2, u)$. **Can’t happen!** $\deg_c(u) \leq 1$.

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Map is injective: if $y_1$ and $y_2$ both map to $(u, \{a, b\})$
then $\text{COL}(y_1, u) = \text{COL}(y_2, u)$. **Can’t happen!** $\deg_c(u) \leq 1$.
So have injection from **infinite** $X - M$ to **finite** $M \times \binom{M}{2}$.
**Contradiction** So $M$ is infinite.
Lemma Let $X$ be infinite. Let $COL : \binom{X}{2} \rightarrow \omega$. Let $d \in \omega$. If for every $x \in X$ and $c \in \omega$, $\deg_c(x) \leq d$ then there is an infinite rainb set. 
Prove on your own.
Can Ramsey Thm for $\mathbb{N}$

**Thm:** For all $COL : \binom{\mathbb{N}}{2} \rightarrow \omega$ there is either

- an infinite homog set,
- an infinite min-homog set,
- an infinite max-homog set, or
- an infinite rainb set.
Proof of Can Ramsey Thm for Graphs

Given $COL : \binom{\mathbb{N}}{2} \to \omega$. We use $COL$ to obtain $COL' : \binom{\mathbb{N}}{3} \to [4]$

We use 3-ary RT. In all of the below $x_1 < x_2 < x_3$. 

Cases 1, 2, 3 are just like in the prior proof.

Case 4 Next slide.
Proof of Can Ramsey Thm for Graphs

Given $COL : \binom{\mathbb{N}}{2} \rightarrow \omega$. We use $COL$ to obtain $COL' : \binom{\mathbb{N}}{3} \rightarrow [4]$.

We use 3-ary RT. In all of the below $x_1 < x_2 < x_3$.

1. If $COL(x_1, x_2) = COL(x_1, x_3)$ then $COL'(x_1 < x_2 < x_3) = 1$. 

Cases 1, 2, 3 are just like in the prior proof. Case 4 Next slide.
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Cases 1,2,3 are just like in the prior proof.

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Proof of Can Ramsey Thm for Graphs

Given $COL : \binom{\mathbb{N}}{2} \rightarrow \omega$. We use $COL$ to obtain $COL' : \binom{\mathbb{N}}{3} \rightarrow [4]$
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1. If $COL(x_1, x_2) = COL(x_1, x_3)$ then $COL'(x_1 < x_2 < x_3) = 1$.
2. If $COL(x_1, x_3) = COL(x_2, x_3)$ then $COL'(x_1 < x_2 < x_3) = 2$.
3. If $COL(x_1, x_2) = COL(x_2, x_3)$ then $COL'(x_1 < x_2 < x_3) = 3$.
4. If none of the above occur then $COL'(x_1 < x_2 < x_3) = 4$.
Cases 1,2,3 are just like in the prior proof.
Case 4
Next slide.
Proof of Can Ramsey Thm for Graphs

Given $COL : (\mathbb{N}^2) \to \omega$. We use $COL$ to obtain $COL' : (\mathbb{N}^3) \to [4]$
We use 3-ary RT. In all of the below $x_1 < x_2 < x_3$.

1. If $COL(x_1, x_2) = COL(x_1, x_3)$ then $COL'(x_1 < x_2 < x_3) = 1$.
2. If $COL(x_1, x_3) = COL(x_2, x_3)$ then $COL'(x_1 < x_2 < x_3) = 2$.
3. If $COL(x_1, x_2) = COL(x_2, x_3)$ then $COL'(x_1 < x_2 < x_3) = 3$.
4. If none of the above occur then $COL'(x_1 < x_2 < x_3) = 4$.

Cases 1,2,3 are just like in the prior proof.
Proof of Can Ramsey Thm for Graphs

Given \( COL : \left( \mathbb{N}^2 \right) \rightarrow \omega \). We use \( COL \) to obtain \( COL' : \left( \mathbb{N}^3 \right) \rightarrow [4] \). We use 3-ary RT. In all of the below \( x_1 < x_2 < x_3 \).

1. If \( COL(x_1, x_2) = COL(x_1, x_3) \) then \( COL'(x_1 < x_2 < x_3) = 1 \).
2. If \( COL(x_1, x_3) = COL(x_2, x_3) \) then \( COL'(x_1 < x_2 < x_3) = 2 \).
3. If \( COL(x_1, x_2) = COL(x_2, x_3) \) then \( COL'(x_1 < x_2 < x_3) = 3 \).
4. If none of the above occur then \( COL'(x_1 < x_2 < x_3) = 4 \).

**Cases 1,2,3** are just like in the prior proof.

**Case 4** Next slide.
Case 4 for all \( x_1 < x_2 < x_3 \)

\( \text{COL}(x_1, x_2) \neq \text{COL}(x_1, x_3) \)
\( \text{COL}(x_1, x_3) \neq \text{COL}(x_2, x_3) \)
\( \text{COL}(x_1, x_2) \neq \text{COL}(x_2, x_3) \)
Case 4 for all $x_1 < x_2 < x_3$

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From this can show that, for all $x$, for all $c$, $\deg_c(x) \leq 1$. 
Case 4 for all $x_1 < x_2 < x_3$

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$COL(x_1, x_2) \neq COL(x_2, x_3)$

From this can show that, for all $x$, for all $c$, $\deg_c(x) \leq 1$. By Lemma on last slide there exists $M \subseteq H$ that is an infinite rainb set.
Better Bounds on Can Ramsey

Using 4-ary proof, 16 colors, bound was:
Better Bounds on Can Ramsey

Using 4-ary proof, 16 colors, bound was:

\[ \text{CR}_2(k) \leq 16^{16^{O(k)}} \]

Using new proof, 3-ary with 4 colors, bound is:
Better Bounds on Can Ramsey

Using 4-ary proof, 16 colors, bound was:

$$\text{CR}_2(k) \leq 16^{16^{O(k)}}$$

Using new proof, 3-ary with 4 colors, bound is:

$$\text{CR}_2(k) \leq 4^{4^{O(k^3)}}$$