**CMSC 752 Homework 10**
**Morally Due Tue April 15, 2025**
**Dead Cat April 17**

1. (50 points)

   **Motivation** As every Hungarian Kindergarten child knows:

   (a) $\exists$ COL: $\binom{[5]}{2} \to [2]$ with NO homog sets. Note that this is the worse case. We wonder about what USUALLY happens.

   (b) For every COL: $\binom{[6]}{2} \to [2]$ there exists two homog sets. Note that this is the worse case. We wonder about what USUALLY happens.

   Problem is on the next page

(a) (Nothing to hand in for this step.) Write a program that will, on input $n \in \mathsf{N}$ generate a coloring of the edges of $K_n$ at random: for each edge, prob of RED is $\frac{1}{2}$ and prob of BLUE is $\frac{1}{2}$. Use adjacency matrices for the graph.

(b) (Nothing to hand in for this step.) Write a program that will, on input a graph $K_n$ that has its edges 2-colored, determines *how many* Homog sets of size 3 are there.

(c) (Nothing to hand in for this step.) Write the following program.

1) Input $n$.

2) For $i = 1$ to 100

i. Generate a graph using the program in Part 1.

ii. Find how many homog triangles there are using the program in Part 2.

iii. Let $A[i]$ be the number of homog triangles.

(d) Calculate the MIN, MAX, and MEAN of the $A[i]$'s.

(e) (This you hand in.)

Write a program that generates the following table (I have made up the numbers).

| n | MIN | MAX | MEAN |
|---|---|---|---|
| 5 | 0 | 5 | 3 |
| 6 | 0 | 5 | 3 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| 40 | 0 | 5 | 2 |

(f) In class we showed that in the worst case there will be $\sim \frac{1}{4}\binom{n}{3}$ mono triangles. Find $A, B, C$ such that the following seems to be true empirically:

*If you choose a coloring at random then*

- *The MIN will be* $\sim A\binom{[n]}{3}$.
- *The MAX will be* $\sim B\binom{[n]}{3}$.
- *The AVG will be* $\sim C\binom{[n]}{3}$.

2. (50 points) Prove the following statement

   *For all $c$ there exists a finite set of grids* $\mathrm{OBS}_c$ *such that*

   *$n \times m$ is $c$-colorable iff $n \times m$ does not contain any element of* $\mathrm{OBS}_c$.

3. (Extra credit)

   (a) Give your name.

   (b) Use Spencer's proof to find reasonably sized graphs $G = (V, E)$ such that

   - $K_4$ is not a subgraph of $G$, and
   - For all COL: $E \to [2]$ there exists a mono triangle.