BILL, RECORD LECTURE!!!!

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One Triangle, Two Triangles

William Gasarch

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Lets Party Like Its 2019

The following is the first theorem in Ramsey Theory:

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Lets Party Like Its 2019

The following is the first theorem in Ramsey Theory: Thm For all 2-col of the edges of K_6 there is a mono K_3 .

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Thm For all 2-cols of edges of K_{12} there are 2 mono K_3 's **Question** Find *n* such that

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- 2. There exists a 2-col of the edges of K_{n-1} that does not have 2 mono K_3 's.

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VOTE (1) n = 12, (2) $9 \le n \le 10$, (3) $6 \le n \le 8$.

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VOTE (1)
$$n = 12$$
, (2) $9 \le n \le 10$, (3) $6 \le n \le 8$.
Answer $n = 6$.

- 1. For all 2-col of the edges of K_6 there are 2 mono K_3 's
- 2. There exists a 2-col of the edges of K_5 that does not have any mono K_3 's.

Proof of K_6 Two Triangles Theorem

Thm For all 2-cols of edges of K_6 there are 2 mono K_3 's

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Proof of K_6 Two Triangles Theorem

Thm For all 2-cols of edges of K_6 there are 2 mono K_3 's **Proof** Let *COL* be a 2-col of the edges of K_6 . Let *R*, *B*, *M*, be the SET of **RED**, **BLUE**, and **MIXED** triangles.

$$|R| + |B| + |M| = \binom{6}{3} = 20.$$

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We show that $|M| \le 18$, so $|R| + |B| \ge 2$.

A Mixed Triangle Has a Vertex Such That



(v₂, v₁) is red, (v₂, v₃) is blue. View this as (v₂, {v₁, v₃}).
 (v₃, v₁) is red, (v₃, v₂) is blue. View this as (v₃, {v₁, v₂}).

Def A **Zan** is an element $(v, \{u, w\}) \in V \times {\binom{V}{2}}$ such that $v \notin \{u, w\}$ and $COL(v, u) \neq COL(v, w)$. ZAN is the set of Zan's.

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Map ZAN to *M* by mapping $(v, \{u, w\})$ to triangle $\{v, u, w\}$. Claim This mapping is exactly 2-to-1.

Def A Zan is an element $(v, \{u, w\}) \in V \times {V \choose 2}$ such that $v \notin \{u, w\}$ and $COL(v, u) \neq COL(v, w)$. ZAN is the set of Zan's. Map ZAN to *M* by mapping $(v, \{u, w\})$ to triangle $\{v, u, w\}$. **Claim** This mapping is exactly 2-to-1. What Zan's map to the triangle:



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 $(v_2, \{v_1, v_3\})$ and $(v_3, \{v_1, v_2\})$.

There is a 2-to-1 map from ZAN to M. Hence

|M| = |ZAN|/2

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Now we want to bound |ZAN|.

There is a 2-to-1 map from ZAN to M. Hence

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Now we want to bound |ZAN|.

Look at vertex v. How many ZAN's use v as their base point?

There is a 2-to-1 map from ZAN to M. Hence

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Now we want to bound |ZAN|.

Look at vertex v. How many ZAN's use v as their base point? Depends on $\deg_{R}(v)$ and $\deg_{B}(v)$.

There is a 2-to-1 map from ZAN to M. Hence

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Now we want to bound |ZAN|.

Look at vertex v. How many ZAN's use v as their base point? Depends on $\deg_{R}(v)$ and $\deg_{B}(v)$.

Thought experiment If $\deg_R(v) = 3$ and $\deg_B(v) = 2$ then how many ZAN's are of the form

 $\{v, \{x, y\}\}$

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$$\{v, \{x, y\}\}$$

x: COL(v, x) = RED. There are $\deg_R(v)$ of them.

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x: COL(v, x) = RED. There are $\deg_{R}(v)$ of them. y: COL(v, y) = BLUE. There are $\deg_{B}(v)$ of them.

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x: COL(v, x) = RED. There are $\deg_R(v)$ of them. y: COL(v, y) = BLUE. There are $\deg_B(v)$ of them. So v contributes $\deg_R(v) \times \deg_B(v)$.

Cases

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Cases

1. v has $\deg_{R}(v) = 5$ or $\deg_{B}(v) = 0$: v contributes 0.

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Cases

- 1. v has deg_R(v) = 5 or deg_B(v) = 0: v contributes 0.
- 2. v has $\deg_{R}(v) = 4$ or $\deg_{B}(v) = 1$: v contributes 4.

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Cases

- 1. v has deg_R(v) = 5 or deg_B(v) = 0: v contributes 0.
- 2. v has deg_R(v) = 4 or deg_B(v) = 1: v contributes 4.
- 3. v has deg_R(v) = 3 or deg_B(v) = 2: v contributes 6. Max.

Cases

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6 vertices, each contribute \leq 6,

Cases

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6 vertices, each contribute \leq 6, so

$$|M| = |ZAN|/2 \le 6 \times 6/2 = 18$$
, so

Cases

- 1. v has deg_R(v) = 5 or deg_B(v) = 0: v contributes 0.
- 2. v has deg_R(v) = 4 or deg_B(v) = 1: v contributes 4.
- v has deg_R(v) = 3 or deg_B(v) = 2: v contributes 6. Max.
 6 vertices, each contribute < 6, so

$$|M| = |ZAN|/2 \le 6 \times 6/2 = 18$$
, so

$$|R| + |B| \ge 20 - |M| \ge 2$$
$$|R| + |B| + |M| = \binom{6}{3} = 20$$

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Map ZAN to M. Map is 2-to-1, so |M| = |ZAN|/2.

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Map ZAN to M. Map is 2-to-1, so |M| = |ZAN|/2.

ZAN is max when each vertex: 3 R and 2 B (or 2 R and 3 B). $|ZAN| \le 6 \times 6 = 36$.

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 $|R| + |B| \ge 20 - |M| \ge 2.$

So there are at least 2 Mono Triangles.

If we 2-color the edges of K_n how many mono K_3 's do we have?



If we 2-color the edges of K_n how many mono K_3 's do we have? **VOTE** (1) ~ n^c for some c < 1, (2) ~ n (3) ~ n^2 , (4) ~ n^3 .

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We do one case: $n \equiv 1 \pmod{2}$.

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We do one case: $n \equiv 1 \pmod{2}$. Let *COL* be a coloring of the edges of K_n .

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We do one case: $n \equiv 1 \pmod{2}$. Let *COL* be a coloring of the edges of K_n . Then degree of each vertex is $n - 1 \equiv 0 \pmod{2}$.

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We find an upper bound on |ZAN|.

To maximize |ZAN| we would, at each vertex, color half of the edges RED and half BLUE.

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To maximize |ZAN| we would, at each vertex, color half of the edges RED and half BLUE. Each vertex contributes $(\frac{n-1}{2})^2$. (Recall $n-1 \equiv 0 \pmod{2}$.)

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To maximize |ZAN| we would, at each vertex, color half of the edges RED and half BLUE. Each vertex contributes $\left(\frac{n-1}{2}\right)^2$. (Recall $n-1 \equiv 0 \pmod{2}$.)

$$|ZAN| \le n \frac{(n-1)^2}{4} = \frac{(n-1)^2 n}{4}$$
 so

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 $\frac{1}{2} = 0 \pmod{2}$

$$|ZAN| \le n \frac{(n-1)^2}{4} = \frac{(n-1)^2 n}{4}$$
 so

$$|M| = |ZAN|/2 \le \frac{(n-1)^2 n}{8}$$

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Recap

$$|M| \leq \frac{(n-1)^2 n}{8}$$

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Recap

$$|M| \leq \frac{(n-1)^2 n}{8}$$

Recall

$$|R| + |B| + |M| = \binom{n}{3} = \frac{n(n-1)(n-2)}{6}$$
 hence

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Recap

$$|M| \leq \frac{(n-1)^2 n}{8}$$

Recall

$$|R| + |B| + |M| = {n \choose 3} = \frac{n(n-1)(n-2)}{6}$$
 hence
 $|R| + |B| = \frac{n(n-1)(n-2)}{6} - |M|$ hence

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Recap

$$|M| \leq \frac{(n-1)^2 n}{8}$$

Recall

$$|R| + |B| + |M| = \binom{n}{3} = \frac{n(n-1)(n-2)}{6} \text{ hence}$$
$$|R| + |B| = \frac{n(n-1)(n-2)}{6} - |M| \text{ hence}$$
$$|R| + |B| \ge \frac{n(n-1)(n-2)}{6} - \frac{(n-1)^2n}{8}$$

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$$|M| \leq \frac{(n-1)^2 n}{8}$$

Recall

$$|R| + |B| + |M| = \binom{n}{3} = \frac{n(n-1)(n-2)}{6} \text{ hence}$$
$$|R| + |B| = \frac{n(n-1)(n-2)}{6} - |M| \text{ hence}$$
$$|R| + |B| \ge \frac{n(n-1)(n-2)}{6} - \frac{(n-1)^2n}{8}$$
$$= \frac{n^3}{24} - \frac{n^2}{4} + \frac{5n}{24}$$

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What About The Other Cases?

We leave the other cases to the reader to both determine the theorem and prove it.

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Can This Be Improved?

The bound is known to be tight.



Coloring that Shows the Bound is Tight

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Go to this pointer: tttalk.pdf

Lets Party Like Its 2019

The following is an early theorem in Ramsey Theory:

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Lets Party Like Its 2019

The following is an early theorem in Ramsey Theory: Thm For all 2-col of the edges of K_{18} there is a mono K_4 .

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Thm For all 2-cols of edges of K_{36} there are 2 mono K_4 's

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Thm For all 2-cols of edges of K_{36} there are 2 mono K_4 's **Smallest** *n* such that \forall 2-col of edges of $K_n \exists$ 2 mono K_4 's?

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Thm For all 2-cols of edges of K_{36} there are 2 mono K_4 's **Smallest** *n* such that \forall 2-col of edges of $K_n \exists$ 2 mono K_4 's? **VOTE** (1) n = 36,

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Thm For all 2-cols of edges of K_{36} there are 2 mono K_4 's **Smallest** *n* such that \forall 2-col of edges of $K_n \exists$ 2 mono K_4 's? **VOTE** (1) n = 36, (2) Some *n*, $19 \le n \le 35$,

Thm For all 2-cols of edges of K_{36} there are 2 mono K_4 's **Smallest** *n* such that \forall 2-col of edges of $K_n \exists$ 2 mono K_4 's? **VOTE** (1) n = 36, (2) Some *n*, $19 \le n \le 35$, (3) n = 18.

Thm For all 2-cols of edges of K_{36} there are 2 mono K_4 's **Smallest** *n* such that \forall 2-col of edges of $K_n \exists 2 \mod K_4$'s? **VOTE** (1) n = 36, (2) Some *n*, $19 \le n \le 35$, (3) n = 18. **Answer** This is really two questions.

Thm For all 2-cols of edges of K_{36} there are 2 mono K_4 's Smallest *n* such that \forall 2-col of edges of $K_n \exists$ 2 mono K_4 's? VOTE (1) n = 36, (2) Some *n*, $19 \le n \le 35$, (3) n = 18. Answer This is really two questions.

1. As posed the answer is n = 18. Piwakoswki and Radziszowski https://www.cs.rit.edu/~spr/PUBL/paper40.pdf showed that for every 2-col of K_{18} there are 9 mono K_4 's.

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2. I will present a math-interesting proof of the following: For all 2-cols of K_{19} there are TWO mono K_4 's.
Thm For all 2-cols of edges of K_{19} there are 2 mono K_4 's

Thm For all 2-cols of edges of K_{19} there are 2 mono K_4 's Assume you have a 2-col of the edges of K_{19} .

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Thm For all 2-cols of edges of K_{19} there are 2 mono K_4 's Assume you have a 2-col of the edges of K_{19} .

List out all subsets of $V = \{1, \ldots, 19\}$ of size R(4) = 18.

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 $A_1, A_2, \ldots, A_{\binom{19}{18}}.$

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Since $|A_i| = R(4)$, each A_i has a mono K_4 .

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Since $|A_i| = R(4)$, each A_i has a mono K_4 . So we get $\binom{19}{18}$ mono K_4 's. So we are almost there.

Thm For all 2-cols of edges of K_{19} there are 2 mono K_4 's Assume you have a 2-col of the edges of K_{19} .

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Thm For all 2-cols of edges of K_{19} there are 2 mono K_4 's Assume you have a 2-col of the edges of K_{19} .

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List out all subsets of $V = \{1, \ldots, 19\}$ of size R(4) = 18.

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List out all subsets of $V = \{1, \ldots, 19\}$ of size R(4) = 18.

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(There are just 19 of these, $A_i = \{1, ..., 19\} - \{i\}$.)

List out all subsets of $V = \{1, \ldots, 19\}$ of size R(4) = 18.

$$A_1, A_2, \ldots, A_{\binom{19}{18}}.$$

(There are just 19 of these, $A_i = \{1, ..., 19\} - \{i\}$.) 1) Find a mono K_4 in A_1 . Say its $\{16, 17, 18, 19\}$.

List out all subsets of $V = \{1, \ldots, 19\}$ of size R(4) = 18.

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(There are just 19 of these, $A_i = \{1, \ldots, 19\} - \{i\}$.)

- 1) Find a mono K_4 in A_1 . Say its $\{16, 17, 18, 19\}$.
- 2) REMOVE all A_i 's that have all of $\{16, 17, 18, 19\}$.

List out all subsets of $V = \{1, \ldots, 19\}$ of size R(4) = 18.

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(There are just 19 of these, $A_i = \{1, ..., 19\} - \{i\}$.) 1) Find a mono K_4 in A_1 . Say its $\{16, 17, 18, 19\}$. 2) REMOVE all A_i 's that have all of $\{16, 17, 18, 19\}$. There are $\binom{19-4}{18-4} = \binom{15}{14} = 15$ of these.

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List out all subsets of $V = \{1, \ldots, 19\}$ of size R(4) = 18.

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(There are just 19 of these, $A_i = \{1, ..., 19\} - \{i\}$.) 1) Find a mono K_4 in A_1 . Say its $\{16, 17, 18, 19\}$. 2) REMOVE all A_i 's that have all of $\{16, 17, 18, 19\}$. There are $\binom{19-4}{18-4} = \binom{15}{14} = 15$ of these. There are $\binom{19}{18} - \binom{15}{14} = 19 - 15 = 4$ left. Call then B_1, B_2, B_3, B_4 . 3) Since B_1 has 18 = R(4) vertices, there is a mono K_4 4) The mono K_4 from A_1 , and the mono K_4 from B are different. Those are our 2 mono K_4 's.

We show that the technique to get 2 mono K_4 's cannot be extended to give 3 mono K_4 's.

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Assume that the first K_4 is $\{16, 17, 18, 19\}$

We show that the technique to get 2 mono K_4 's cannot be extended to give 3 mono K_4 's.

Assume that the first K_4 is $\{16, 17, 18, 19\}$

Assume that the second K_4 is $\{12, 13, 14, 15\}$

We show that the technique to get 2 mono K_4 's cannot be extended to give 3 mono K_4 's.

- Assume that the first K_4 is $\{16, 17, 18, 19\}$
- Assume that the second K_4 is $\{12, 13, 14, 15\}$
- So we have removed all $A \subseteq \{1, \ldots, 19\}$ of size 18 where

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We show that the technique to get 2 mono K_4 's cannot be extended to give 3 mono K_4 's. Assume that the first K_4 is $\{16, 17, 18, 19\}$ Assume that the second K_4 is $\{12, 13, 14, 15\}$ So we have removed all $A \subseteq \{1, \ldots, 19\}$ of size 18 where A has all of $\{16, 17, 18, 19\}$, or

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We show that the technique to get 2 mono K_4 's cannot be extended to give 3 mono K_4 's. Assume that the first K_4 is {16, 17, 18, 19} Assume that the second K_4 is {12, 13, 14, 15} So we have removed all $A \subseteq \{1, \ldots, 19\}$ of size 18 where A has all of {16, 17, 18, 19}, or A has all of {12, 13, 14, 15}.

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We show that the technique to get 2 mono K_4's cannot be
extended to give 3 mono K_4's.
Assume that the first K_4 is {16,17,18,19}
Assume that the second K_4 is {12,13,14,15}
So we have removed all A \subseteq \{1, \ldots, 19\} of size 18 where
A has all of {16,17,18,19}, or
A has all of {12,13,14,15}.
Whats left?
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We show that the technique to get 2 mono K_4 's cannot be extended to give 3 mono K_4 's.

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Assume that the second K_4 is $\{12, 13, 14, 15\}$

So we have removed all $A \subseteq \{1, \ldots, 19\}$ of size 18 where

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A has all of \{16, 17, 18, 19\}, or
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A has all of \{12, 13, 14, 15\}.
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Whats left?All $A \subseteq \{1, \ldots, 19\}$ of size 18 that are missing at least one of $\{16, 17, 18, 19\}$ and at least one of $\{12, 13, 14, 15\}$.

We show that the technique to get 2 mono K_4 's cannot be extended to give 3 mono K_4 's.

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If A \subseteq \{1, \ldots, 19\} is missing two elements it is of size 17.
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We show that the technique to get 2 mono K_4 's cannot be extended to give 3 mono K_4 's.

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Assume that the first K_4 is \{16, 17, 18, 19\}
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Whats left?All $A \subseteq \{1, ..., 19\}$ of size 18 that are missing at least one of $\{16, 17, 18, 19\}$ and at least one of $\{12, 13, 14, 15\}$. If $A \subseteq \{1, ..., 19\}$ is missing two elements it is of size 17.

Hence there might be no mono K_4 's.

We show that the technique to get 2 mono K_4 's cannot be extended to give 3 mono K_4 's.

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If $A \subseteq \{1, \ldots, 19\}$ is missing two elements it is of size 17.

Hence there might be no mono K_4 's.

Note We only showed that the **proof** cannot be extended. As noted above any 2-col of K_{18} has 9 mono K_4 's.

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 \forall 2-col of $K_n \exists$ 3 mono K_4 's.



 \forall 2-col of $K_n \exists$ 3 mono K_4 's.

List out all subsets of $V = \{1, \ldots, n\}$ of size R(4) = 18.

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List out all subsets of $V = \{1, \ldots, n\}$ of size R(4) = 18.

$$A_1, A_2, \ldots, A_{\binom{n}{18}}$$

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 \forall 2-col of $K_n \exists$ 3 mono K_4 's.

List out all subsets of $V = \{1, \ldots, n\}$ of size R(4) = 18.

$$A_1, A_2, \ldots, A_{\binom{n}{18}}.$$

1) Find a mono K_4 in A_1 . Say its $\{x_1, x_2, x_3, x_4\}$.

 \forall 2-col of $K_n \exists$ 3 mono K_4 's.

List out all subsets of $V = \{1, \ldots, n\}$ of size R(4) = 18.

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Find a mono K₄ in A₁. Say its {x₁, x₂, x₃, x₄}.
 REMOVE all A_i's that have all of {x₁, x₂, x₃, x₄}.

 \forall 2-col of $K_n \exists$ 3 mono K_4 's.

List out all subsets of $V = \{1, \ldots, n\}$ of size R(4) = 18.

$$A_1, A_2, \ldots, A_{\binom{n}{18}}.$$

1) Find a mono K_4 in A_1 . Say its $\{x_1, x_2, x_3, x_4\}$. 2) REMOVE all A_i 's that have all of $\{x_1, x_2, x_3, x_4\}$. $\binom{n-4}{18-4} = \binom{n-4}{14}$ of these. There are $\binom{n}{18} - \binom{n-4}{14}$ left.
\forall 2-col of $K_n \exists$ 3 mono K_4 's.

List out all subsets of $V = \{1, \ldots, n\}$ of size R(4) = 18.

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1) Find a mono K_4 in A_1 . Say its $\{x_1, x_2, x_3, x_4\}$. 2) REMOVE all A_i 's that have all of $\{x_1, x_2, x_3, x_4\}$. $\binom{n-4}{18-4} = \binom{n-4}{14}$ of these. There are $\binom{n}{18} - \binom{n-4}{14}$ left. 3) Find a mono K_4 in one of the sets left. Say its $\{y_1, y_2, y_3, y_4\}$.

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 \forall 2-col of $K_n \exists$ 3 mono K_4 's.

List out all subsets of $V = \{1, \ldots, n\}$ of size R(4) = 18.

$$A_1, A_2, \ldots, A_{\binom{n}{18}}.$$

1) Find a mono K_4 in A_1 . Say its $\{x_1, x_2, x_3, x_4\}$. 2) REMOVE all A_i 's that have all of $\{x_1, x_2, x_3, x_4\}$. $\binom{n-4}{18-4} = \binom{n-4}{14}$ of these. There are $\binom{n}{18} - \binom{n-4}{14}$ left. 3) Find a mono K_4 in one of the sets left. Say its $\{y_1, y_2, y_3, y_4\}$. 4) REMOVE all A_i 's that have all of $\{y_1, y_2, y_3, y_4\}$.

 \forall 2-col of $K_n \exists$ 3 mono K_4 's.

List out all subsets of $V = \{1, \ldots, n\}$ of size R(4) = 18.

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 \forall 2-col of $K_n \exists$ 3 mono K_4 's.

List out all subsets of $V = \{1, \ldots, n\}$ of size R(4) = 18.

$$A_1, A_2, \ldots, A_{\binom{n}{18}}.$$

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Want 3 Mono K₄'s (cont)

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Need

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$$\binom{n}{18} - 2\binom{n-4}{14} \ge 1$$

Need

$$\binom{n}{18} - 2\binom{n-4}{14} \ge 1$$
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 $n(n-1)(n-2)(n-3) > 2 \times 18 \times 17 \times 16 \times 15$

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n	n(n-1)(n-2)(n-3)
19	93024
20	116280
21	143640
22	175560

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Thm \forall 2-cols of the edges of $K_{22} \exists$ 3 mono K_4 's.

Want: If Color Edges of K_n Then How many Triangles?

This will be a HW.

