BILL, RECORD LECTURE!!!!

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Monochromatic C_4

Exposition by William Gasarch

Credit Where Credit is Due

The main theorem in these slides was proven by

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Here is a link
`https://www.cs.umd.edu/~gasarch/TOPICS/eramsey/c4.pdf`

When Do You Get A Mono C_4?

Thm For all $\mathrm{COL}\colon \binom{[18]}{2} \rightarrow [2]$ there exists a mono C_4.

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Since $R(4) = 18$ there is a mono K_4, hence a mono C_4.

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Vote
1) $R(C_4) = 18$.

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1) $R(C_4) = 18$.
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Answer on the next page.

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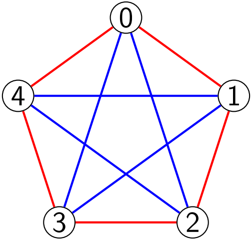

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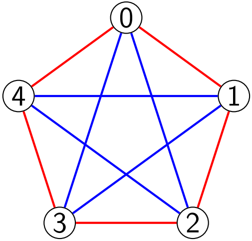

Note: There is a mono C_5 but not a mono C_4.

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Let $\mathrm{COL}\colon \binom{[6]}{2} \to [2]$.

We know that there is a mono triangle.

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$\exists$ a mono triangle. We assume **R** and on vertices $\{1, 2, 3\}$.

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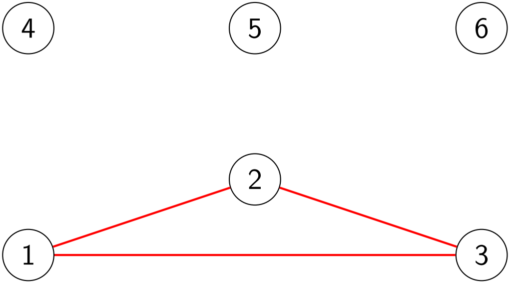

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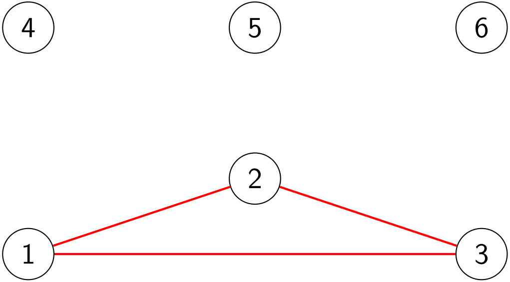

We view $\{1, 2, 3\}$ and $\{4, 5, 6\}$ as the sides of a bipartite graph.

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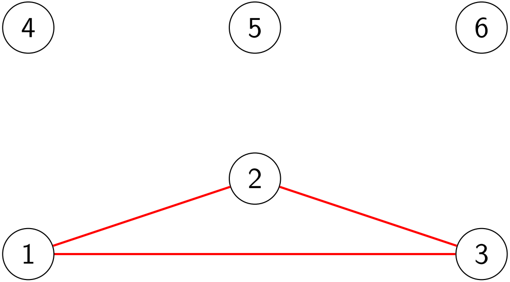

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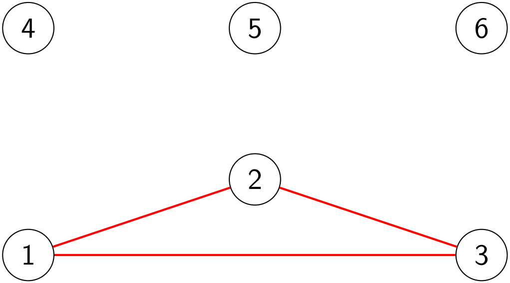

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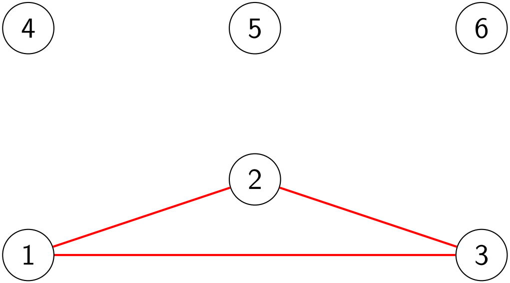

We view $\{1, 2, 3\}$ and $\{4, 5, 6\}$ as the sides of a bipartite graph.
$\deg_{\mathbf{R}}(1)$ will mean the number of **R** edges between 1 and $\{4, 5, 6\}$.
$\deg_{\mathbf{R}}(4)$ will mean the number of **R** edges between 4 and $\{1, 2, 3\}$.
Generalize to $\deg_{\mathbf{R}}(v)$ and $\deg_{\mathbf{B}}(v)$.

R Degree of $4, 5, 6$

$\mathbf{R}$ Degree of $4, 5, 6$

If $\exists v \in \{4, 5, 6\}$, $\deg_{\mathbf{R}}(v) \geq 2$ then get C_4:

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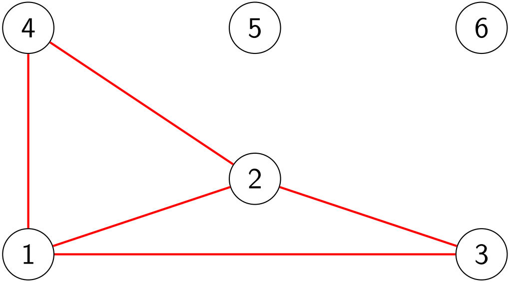

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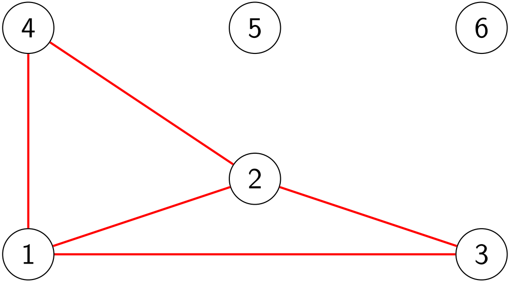

C_4: $4 - 1 - 3 - 2 - 4$.

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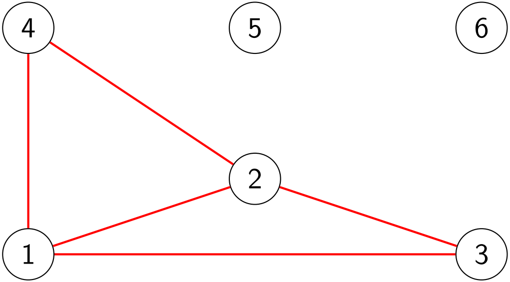

C_4: $4 - 1 - 3 - 2 - 4$.

Note: $(\forall v \in \{4, 5, 6\})[\deg_{\mathbf{B}}(v) \geq 2]$.

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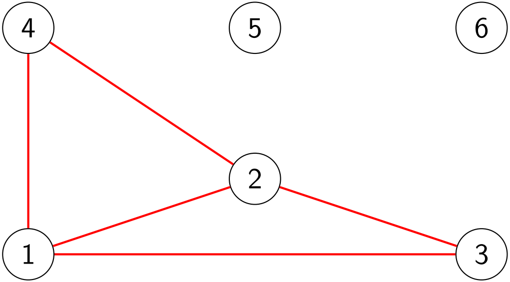

C_4: $4 - 1 - 3 - 2 - 4$.

Note: $(\forall v \in \{4,5,6\})[\deg_{\mathbf{B}}(v) \ge 2]$.
We will show that for $(\forall v \in \{4,5,6\})[\deg_{\mathbf{B}}(v) = 2]$.

B Degree of $4, 5, 6$

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If $(\exists v \in \{4, 5, 6\})[\deg_{\mathbf{B}}(v) = 3]$ then get C_4:

$\mathbf{B}$ Degree of $4, 5, 6$

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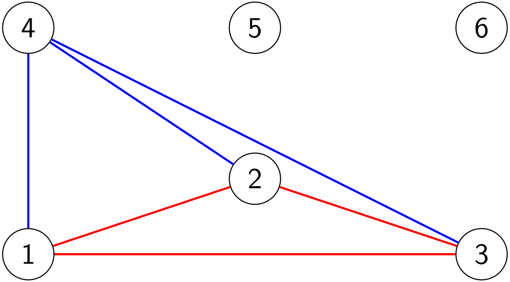

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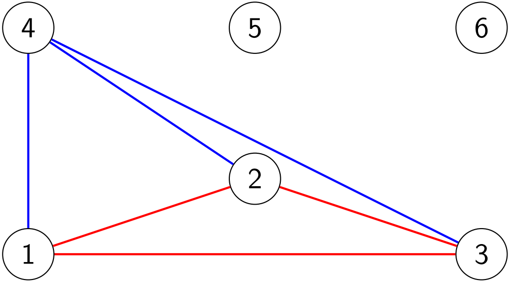

Recall that $\deg_{\mathbf{B}}(5) \geq 2$.

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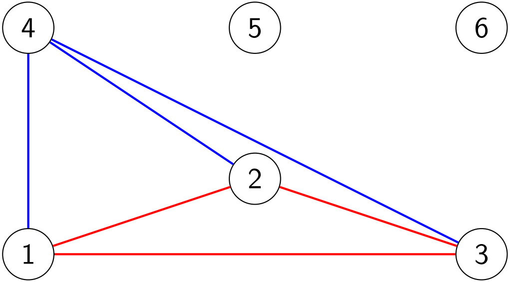
If $(\exists v \in \{4,5,6\})[\deg_{\mathbf{B}}(v) = 3]$ then get C_4:



Recall that $\deg_{\mathbf{B}}(5) \ge 2$.
If $\mathrm{COL}(5,1) = \mathrm{COL}(5,2) = $ **B** then C_4: $5 - 1 - 4 - 2 - 5$.

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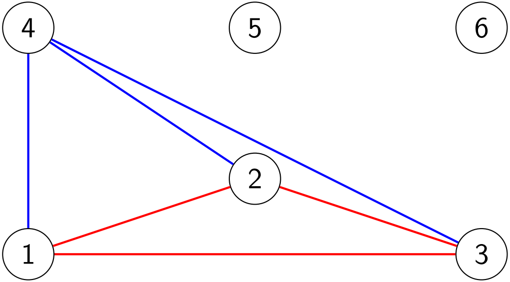

Recall that $\deg_{\mathbf{B}}(5) \geq 2$.

If $\mathrm{COL}(5,1) = \mathrm{COL}(5,2) = \mathbf{B}$ then C_4: $5 - 1 - 4 - 2 - 5$.

If $\mathrm{COL}(5,2) = \mathrm{COL}(5,3) = \mathbf{B}$ then C_4: $5 - 3 - 4 - 2 - 5$.

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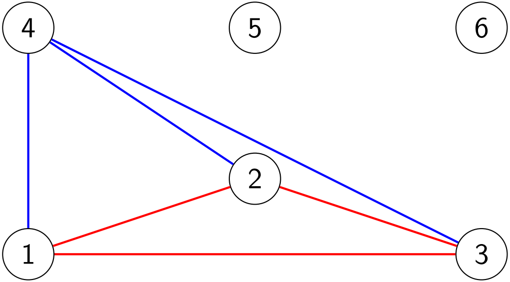

Recall that $\deg_{\mathbf{B}}(5) \ge 2$.

If $\mathrm{COL}(5,1) = \mathrm{COL}(5,2) = \mathbf{B}$ then C_4: $5 - 1 - 4 - 2 - 5$.

If $\mathrm{COL}(5,2) = \mathrm{COL}(5,3) = \mathbf{B}$ then C_4: $5 - 3 - 4 - 2 - 5$.

If $\mathrm{COL}(5,1) = \mathrm{COL}(5,3) = \mathbf{B}$ then C_4: $5 - 1 - 4 - 3 - 5$.

Recap

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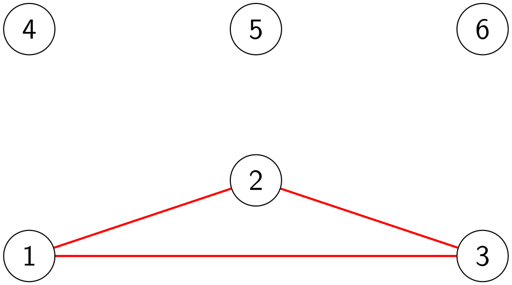

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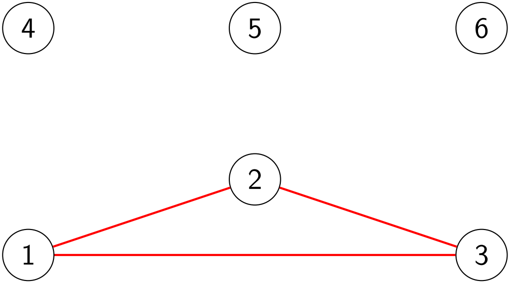
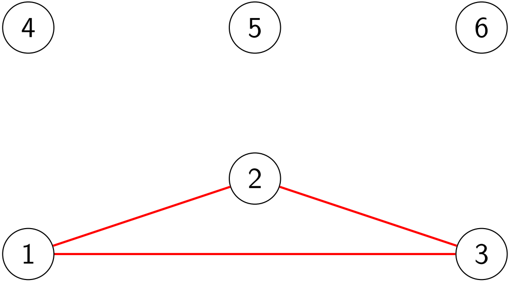


For $v \in \{4, 5, 6\}$, $\deg_{\mathbf{R}}(v) = 1$ and $\deg_{\mathbf{B}}(v) = 2$.

Recap



For $v \in \{4, 5, 6\}$, $\deg_{\mathbf{R}}(v) = 1$ and $\deg_{\mathbf{B}}(v) = 2$.

We now look at the degrees of $1, 2, 3$.

For $v \in \{\mathbf{1}, \mathbf{2}, \mathbf{3}\}$**,** $\deg_{\mathbf{R}}(v) = 1$

For $v \in \{1, 2, 3\}$, $\deg_{\mathbf{R}}(v) = 1$

If $\exists v \in \{1, 2, 3\}$ $\deg_{\mathbf{R}}(v) \geq 2$ then $\mathbf{C_4}$ or $\mathbf{C_4}$.

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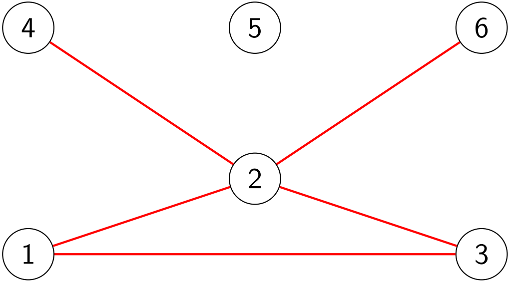

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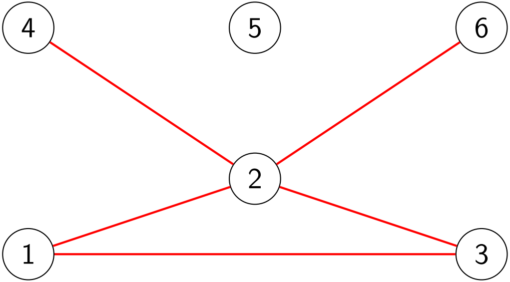

If $\mathrm{COL}(1,4) = \mathbf{R}$ then C_4: $1 - 4 - 2 - 3 - 1$.

For $v \in \{1, 2, 3\}$, $\deg_{\mathbf{R}}(v) = 1$

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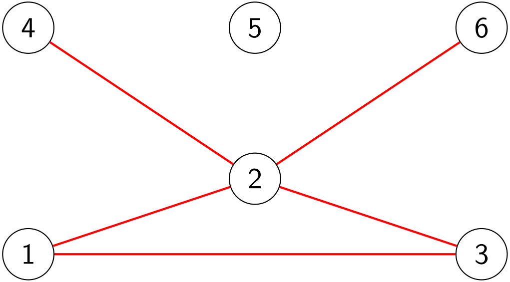

If $\mathrm{COL}(1, 4) = \mathbf{R}$ then C_4: $1 - 4 - 2 - 3 - 1$.
If $\mathrm{COL}(3, 6) = \mathbf{R}$ then C_4: $3 - 6 - 2 - 1 - 3$.

For $v \in \{1,2,3\}$, $\deg_{\mathbf{R}}(v) = 1$

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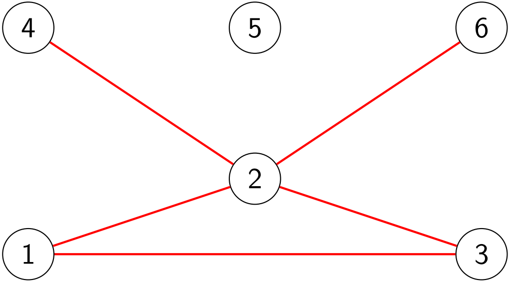

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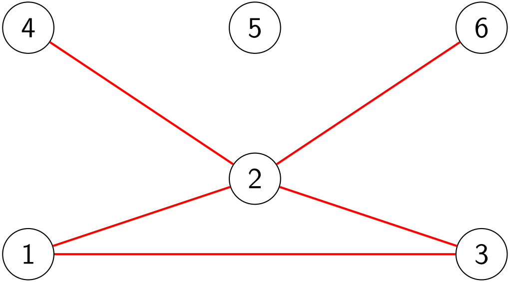

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For $v \in \{1,2,3\}$, $\deg_{\mathbf{R}}(v) = 1$

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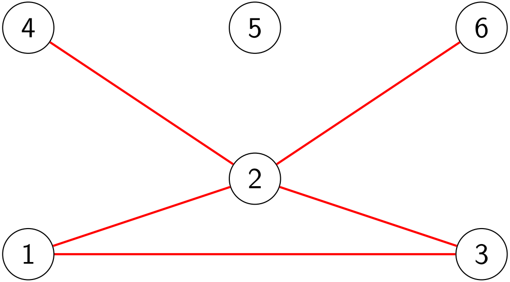

If $\mathrm{COL}(1,4) = \mathbf{R}$ then C_4: $1 - 4 - 2 - 3 - 1$.
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If $\mathrm{COL}(1,6) = \mathbf{R}$ then C_4: $1 - 6 - 2 - 3 - 1$.
If all of those edges are $\mathbf{B}$ then C_4: $1 - 4 - 3 - 6 - 1$.

Red Degree Recap

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- $(\forall v \in \{4, 5, 6\}[\deg_{\mathbf{R}}(v) = 1]$.

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  - $\mathrm{COL}(1,4) = \mathrm{COL}(2,5) = \mathrm{COL}(4,6) =$ **R**

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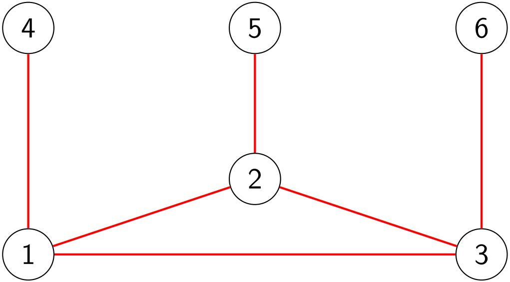
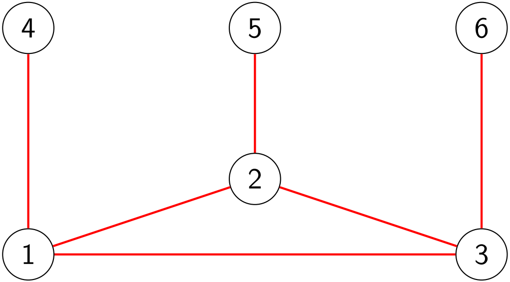
- Hence we can assume
  - $\mathrm{COL}(1,4) = \mathrm{COL}(2,5) = \mathrm{COL}(4,6) =$ **R**
  - All other edges between $\{1,2,3\}$ and $\{4,5,6\}$ are **B**. (We will find some other edges that must be **B**.)

What We Know: R

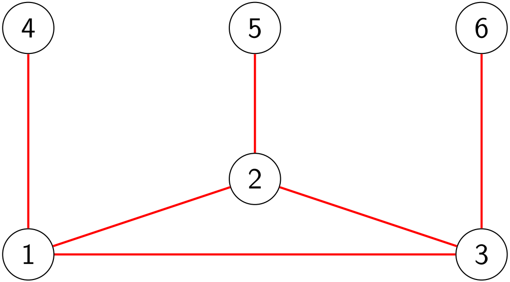


What We Know: R



All edges between $\{1, 2, 3\}$ and $\{4, 5, 6\}$ not shown are **B**.

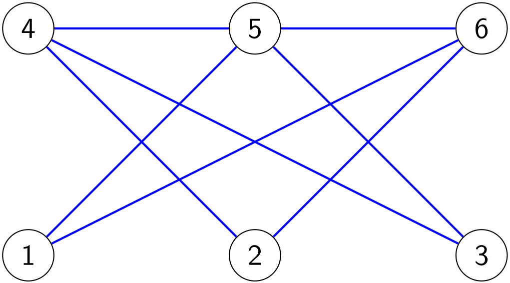
What We Know: R



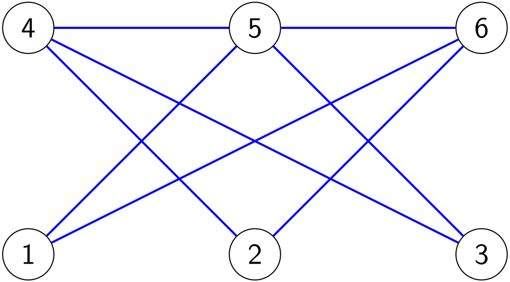
All edges between $\{1, 2, 3\}$ and $\{4, 5, 6\}$ not shown are **B**.

Clearly $\mathrm{COL}(4, 5) = \mathrm{COL}(5, 6) =$ **B**.

What We Know: B

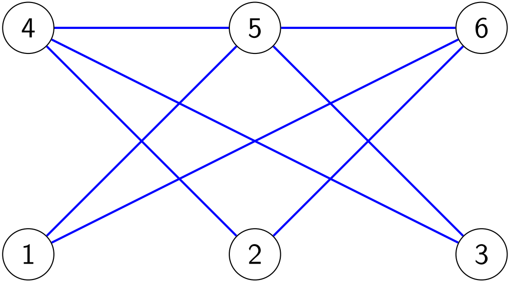


What We Know: B



C_4: $2 - 4 - 5 - 6 - 2$.

What We Know: B



C_4: 2 – 4 – 5 – 6 – 2.

DONE!