BILL, RECORD LECTURE!!!!

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Finite Ramsey Theorem For Graphs

Exposition by William Gasarch

February 4, 2025

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Examples of The First Few Steps of The Construction

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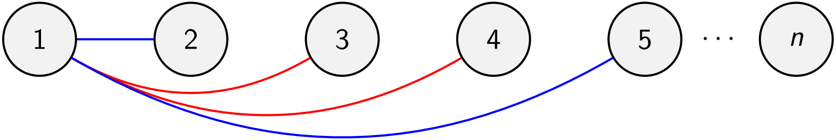

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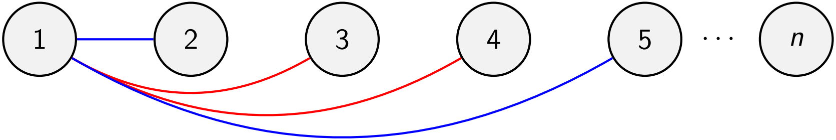

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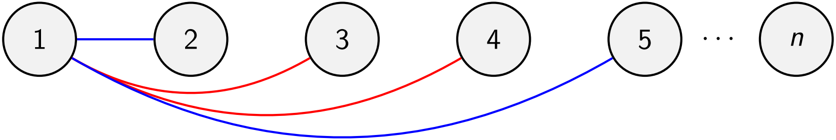

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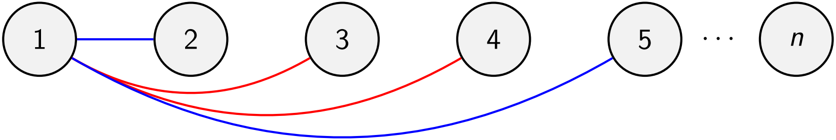

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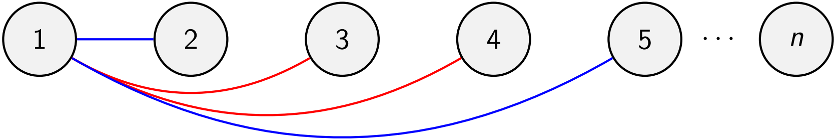

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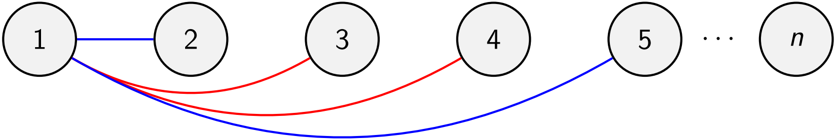

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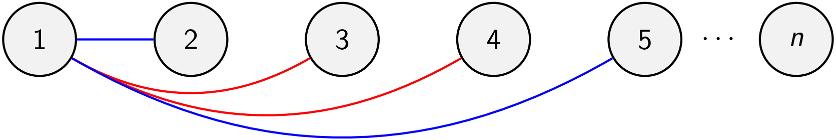

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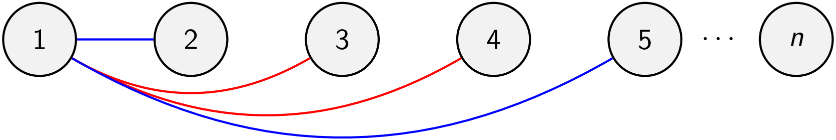

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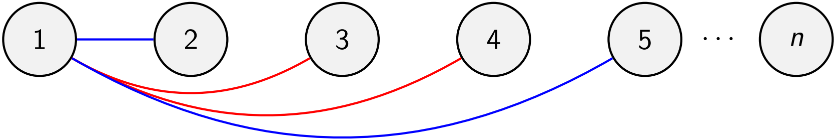

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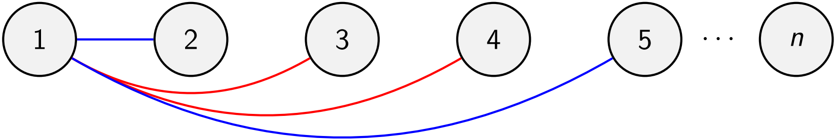

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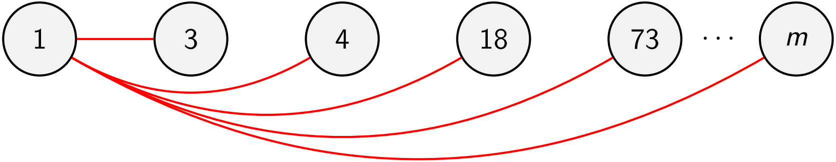

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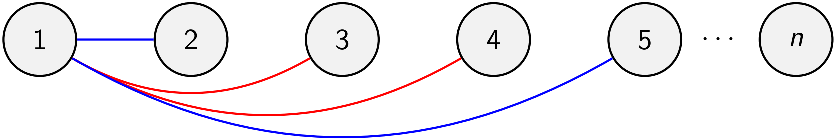

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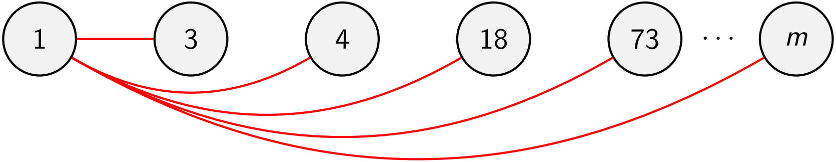

We Omit 1 from future pictures but its **Still Alive and Well.**
`https://www.youtube.com/watch?v=8--jVqaU-G8`.

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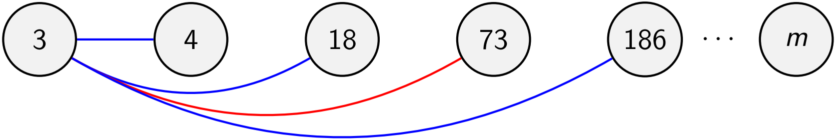

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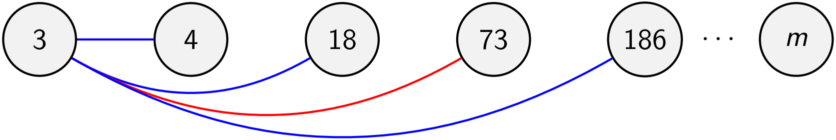

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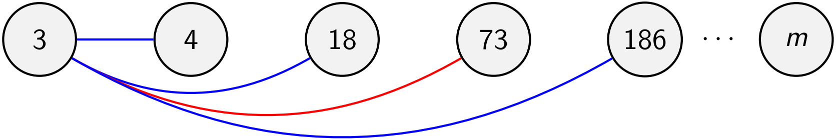

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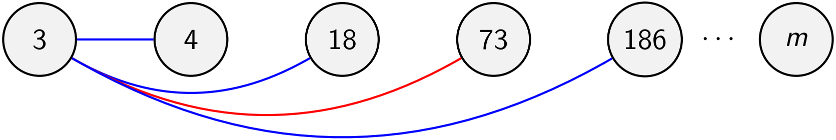

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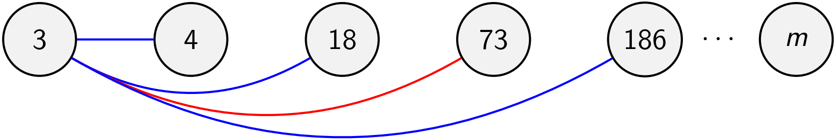

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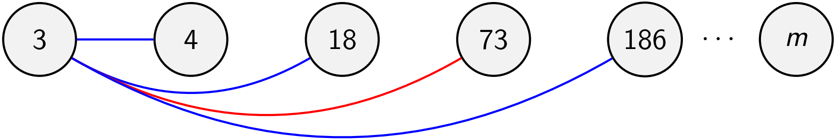

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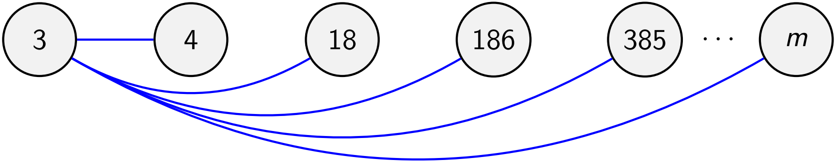

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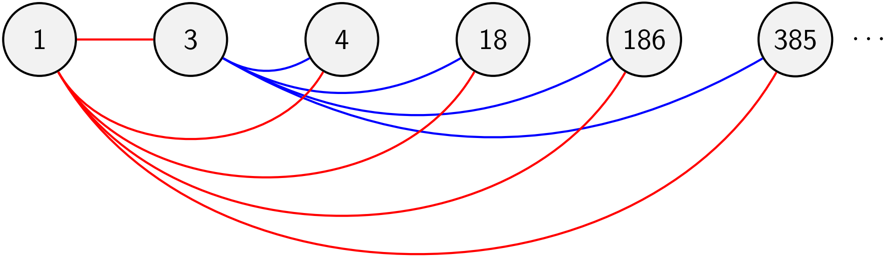

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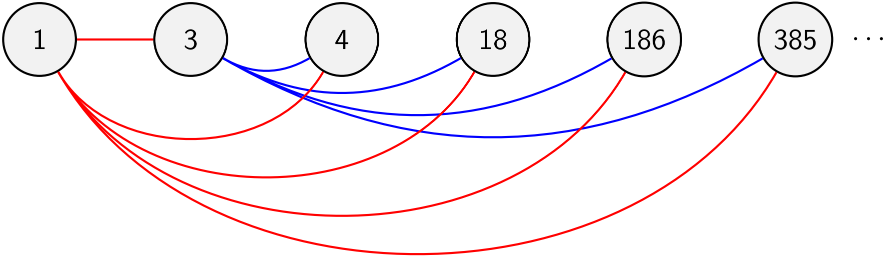

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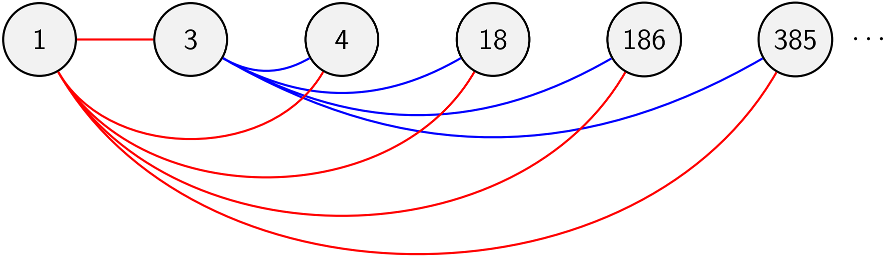

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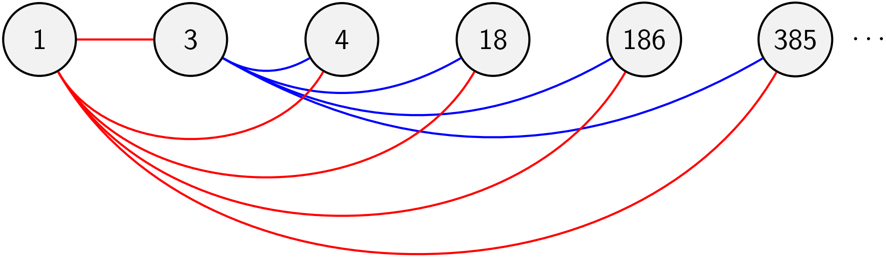

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After s stages still have $n/2^s$ Nodes In Play.

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$H_2 = \{x_1\} \cup \{y \in H_1 \colon \mathrm{COL}(x_1, y) = c_1\}$. Note $|H_2| \geq n/2$.
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Assume H_s, x_s, c_s are defined.
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But the ... is NOT infinite. Where to stop? See next slide

The Size of X

We will see later than we want $|X| \geq 2k - 1$.

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It suffices to take $n \geq 2^{2k-1}$.

The Coloring of the Nodes

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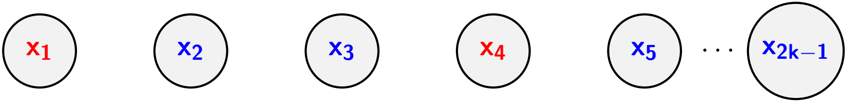

All of the edges from x_1 to the right are **R**.

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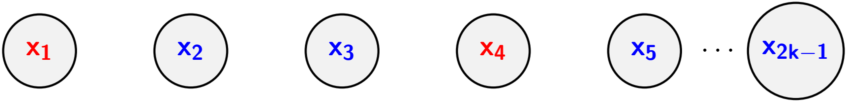

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All of the edges from x_2 to the right are **B**.

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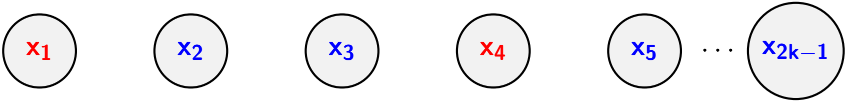

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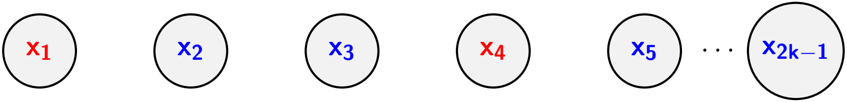

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All of the edges from x_4 to the right are **R**.

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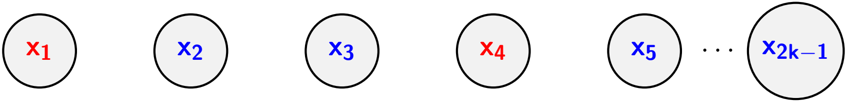

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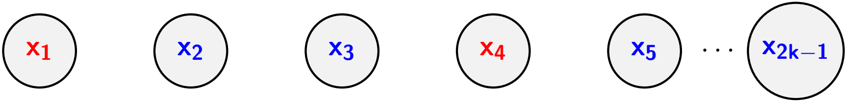

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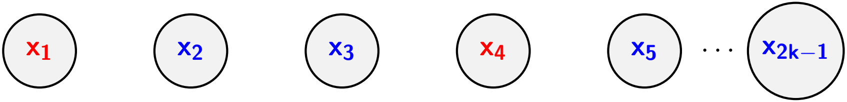

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What do you think our next step is?

Some Color Appears k Times

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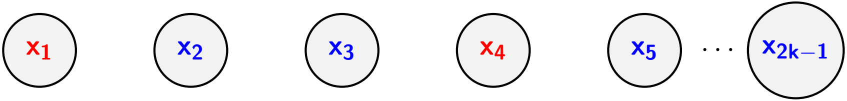

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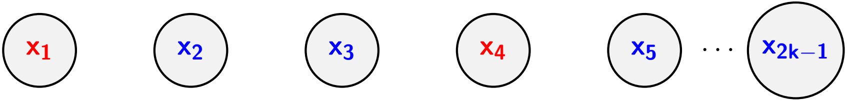

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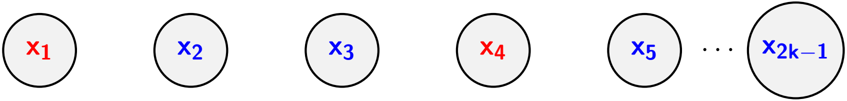

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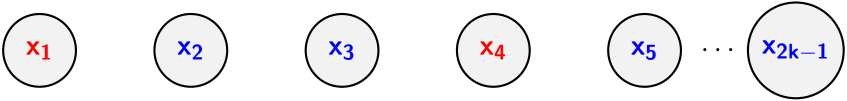

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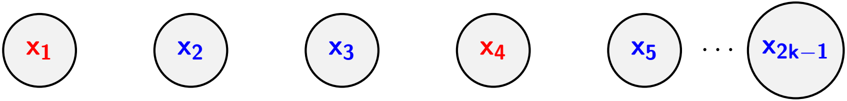

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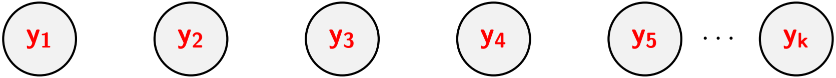

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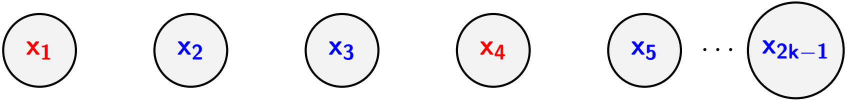

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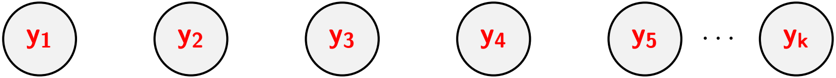

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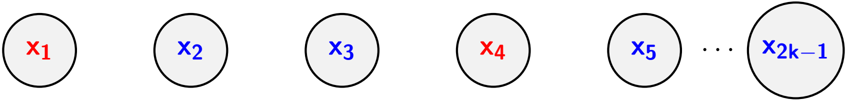

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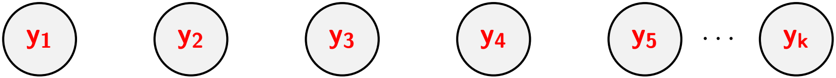

All of the edges from $\mathbf{y_s}$ to the right are **R**.

H is clearly a homog set!

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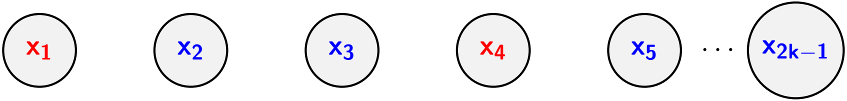

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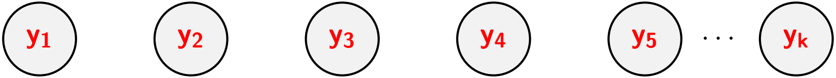

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DONE!

Variants Of
The
Finite Ramsey Theorem

More Colors

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Thm For all $c \in \mathbb{N}$, for all k, there exists $n = R(k,c)$ such that for all $\mathrm{COL}\colon \binom{[n]}{2} \rightarrow [c]$ $\exists$ a homog set of size $\geq k$.

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This is easy to prove using the same technique we used for the $c = 2$ case. It may be a HW to get a bound on $R(k, c)$.

What About Hypergraphs?

Def A a-**hypergraph** is (V, E) where $E \subseteq \binom{V}{a}$.

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Def A ***a*-hypergraph** is (V, E) where $E \subseteq \binom{V}{a}$.

We will define homog sets for colorings of the edges of an a-hypergraph and prove **Finite Ramsey Thm for *a*-hypergraphs.** In a later lecture.

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In later talks we will do the following:

1. Get better, but still exponential, upper bounds on $R(k)$.
2. Obtain lower bound on $R(k)$.