# **BILL, RECORD LECTURE!!!!**

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# The Large Ramsey Theorem

**Exposition by William Gasarch** 

February 18, 2025

Let  $a, n \in \mathbb{N}$ . Let A be a set. A can be finite or infinite.

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- 4.  $\binom{A}{a}$  is the set of all a-sized subsets of A.

Let COL:  $\binom{A}{2} \rightarrow [2]$ . A set  $H \subseteq A$  is **homogenous** if COL restricted to  $\binom{H}{2}$  is constant. (From now on **homog**.)

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 $\{5, 10, 12, 17, 20\}$  is NOT large.

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#### **Examples**

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\{1,2,10\} is large. \{5,10,12,17,20\} is NOT large. \{20,30,40,50,60,70,80,90,100\} is NOT large. \{5,30,40,50,60,70,80,90,100\} is large. \{101,\ldots,190\} is NOT large.
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This is a stupid thm. Discuss.  $\{1,2\}$  is always a large homogenous set. How to change the statement so thats its not stupid?

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Example k=10. Lets say n=990 works. Then \forall \text{COL}: \binom{\{10,\dots,1000\}}{2} \rightarrow [2] \ \exists H \ \text{homog such that} \min(H)=10 \ \text{and} \ |H| \geq 11, \ \text{or} \min(H)=11 \ \text{and} \ |H| \geq 12, \ \text{or} \vdots \qquad \vdots \qquad \vdots \min(H)=499 \ \text{and} \ |H| \geq 500, \ \text{or}
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# Proof of the Large Ramsey Thm From The Infinite Ramsey Thm

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$$(\exists k)(\forall n)(\exists \mathrm{COL}\colon \binom{\{k,\dots,k+n\}}{2})\to [2]$$
 with no Large homog set).

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Say k=182. There is a coloring of  $\binom{\{182,\dots,182+10^{100}\}}{2}$  with no large homog set.

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Say k=182. There is a coloring of  $\binom{\{182,\dots,182+10^{100}\}}{2}$  with no large homog set. That seems unlikely.

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 $COL_2: {\{k,k+1,k+2\} \choose 2} \rightarrow [2]$  with no Large homog set.

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We use COL_0, COL_1, \ldots to form
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We will use the **Inf Ramsey Theory** to get a contradiction.

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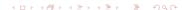
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Proof of Large Ramsey: The following is a large homog set:

$$H_{\text{LargeSet}} = \{x_1 < \dots < x_{x_1+1}\}.$$

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There is an L such that COL restricted to  $H_{\text{LargeSet}}$  is some COL $_L$ .

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There is an L such that COL restricted to  $H_{LargeSet}$  is some COL<sub>L</sub>.

This is a contradiction since  $COL_L$  has no large homog sets.



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**Thm** For all k there exists n = LR(k) such that for all COL:  $\binom{\{k,\dots,k+n\}}{2} \to [2]$  there exists a Large homog set. **BILL:** So we have proven that, for all k, there is an n = LR(k). **STUDENT:** Great! what is LR(10)?

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**STUDENT:** Great! what is LR(10)?

**BILL:** We showed LR(10) exists by showing there is SOME n such that for all COL:  $\binom{\{10,\dots,10+n\}}{2} \rightarrow [2]$  there is a large homog set.

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**STUDENT:** Surely the proof gives an upper bound on LR(10)!

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BILL: The proof is nonconstructive. And don't call me Shirley.

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STODENT: Great: What is LN(10):

**BILL:** We showed LR(10) exists by showing there is SOME n such that for all  $COL: \binom{\{10,\dots,10+n\}}{2} \to [2]$  there is a large homog set.

**STUDENT:** Surely the proof gives an upper bound on LR(10)!

**BILL:** The proof is nonconstructive. And don't call me Shirley.

**STUDENT:** Dagnabbit! I want a bound on LR(10)!

**BILL:** You want an upper bound on the factorial of LR(10)?

**Thm** For all k there exists n = LR(k) such that for all COL:  $\binom{\{k,\dots,k+n\}}{2} \to [2]$  there exists a Large homog set. **BILL:** So we have proven that, for all k, there is an n = LR(k). **STUDENT:** Great! what is LR(10)? **BILL:** We showed LR(10) exists by showing there is SOME n such that for all COL:  $\binom{\{10,\dots,10+n\}}{2} \to [2]$  there is a large homog set. **STUDENT:** Surely the proof gives an upper bound on LR(10)! **BILL:** The proof is nonconstructive. And don't call me Shirley. **STUDENT:** Dagnabbit! I want a bound on LR(10)! **BILL:** You want an upper bound on the factorial of LR(10)? **STUDENT:** No you muffinhead, I want a bound on LR(10) and I feel strongly about it.

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**STUDENT:** You are telling the same jokes twice, with **Shirley** and **Factorial**.

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Summary of what we have done and what might be on HW: Thm  $(\forall a)(\forall k)(\exists n)\forall \text{COL}: \binom{k,\dots,k+n}{a} \rightarrow [2]$  there exists a large homog set.

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- $a \ge 3$ : Easy HW. No bounds no  $LR_a(k)$ .

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Answer on next slide.

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Discuss: Is LR natural?