Poly VDW Theorem Cheat Sheet I
Statment of Poly VDW and Lemmas We Proved

Theorem 0.1. Poly VDW Theorem *Let* $p_1, \ldots, p_k \in \mathsf{Z}[x]$ *be such that* $p_i(0) = 0$. *Let* $c \in \mathsf{N}$. *Then* $\exists\ W = W(p_1, \ldots, p_k; c)$ *such that for all* $\mathrm{COL}\colon [W] \to [c]$ $\exists a, d$, *such that*

$$a, a + p_1(d), \ldots, a + p_k(d) \text{ same col.}$$

Lemma 0.2. *Let* $c \in \mathsf{N}$.
$(\forall r)(\exists U = U(r)(\forall \mathrm{COL}\colon [U] \to [c])(\exists a, d_1, \ldots, d_r)$ *such that*

either

$(\exists a, d)[\mathrm{COL}(a) = \mathrm{COL}(a + d^2)]$, *or*

$(\exists a, d_1, \ldots, d_r)[\mathrm{COL}(a), \mathrm{COL}(a + d_1^2), \ldots, \mathrm{COL}(a + d_r^2)$ *all different*].

Lemma 0.3. *Let* $c \in \mathsf{N}$.
$(\forall r)(\exists U = U(r)(\forall \mathrm{COL}\colon [U] \to [c])(\exists a, d_1, \ldots, d_r)$ *such that*

either

$(\exists a, d)[\mathrm{COL}(a) = \mathrm{COL}(a + d^2) = \mathrm{COL}(a + d^2 + d)$ *or*

$(\exists a, d_1, \ldots, d_r)$
$\mathrm{COL}(a)$
$\mathrm{COL}(a + d_1^2) = \mathrm{COL}(a + d_1^2 + d_1)$
$\mathrm{COL}(a + d_2^2) = \mathrm{COL}(a + d_2^2 + d_2)$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$
$\mathrm{COL}(a + d_r^2) = \mathrm{COL}(a + d_r^2 + d_r)$
And all of the above are different colors.

Lemma 0.4. *Let* $c \in \mathsf{N}$.
$(\forall r)(\exists U = U(r)(\forall \mathrm{COL}\colon [U] \to [c])(\exists a, d_1, \ldots, d_r)$ *such that*
either

$(\exists a, d)[\mathrm{COL}(a) = \mathrm{COL}(a + d) = \mathrm{COL}(a + d^2)$ *or*

$(\exists a, d_1, \ldots, d_r)$
$\mathrm{COL}(a)$
$\mathrm{COL}(a + d_1) = \mathrm{COL}(a + d_1^2)$
$\mathrm{COL}(a + d_2) = \mathrm{COL}(a + d_2^2)$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$
$\mathrm{COL}(a + d_r) = \mathrm{COL}(a + d_r^2)$
And all of the above are different colors.