Overview: Ramsey Theory is a branch of combinatorics having to do with colorings and patterns. Here is a sample theorem: for all 2-colorings of the natural numbers there exists arbitrarily long monochromatic arithmetic sequences (arithmetic sequences are equally spaced, like 11,14,17,20,23,27). In this course we state and prove many such theorems and also “apply” them.

1. The infinite and finite Erdos-Szekeres theorem on monotone sequences APPLICATION to Lower bounds on Branching Programs.

2. The infinite Ramsey Theorem APPLICATION to Proving Programs correct, well-quasi ordering, Logic (Ramsey’s original motivation!). Canonical Ramsey Theorem. APPLICATION to lower bounds on Parallel Models of Computation.

3. The finite Ramsey Theorems Upper and lower bounds on the Ramsey Numbers. APPLICATIONS to lower bounds on various models of computation, the Erdos-Szekeres theorem in geometry, Sociology, History.

4. The Large Ramsey Theorem APPLICATION: A natural(?) example of something in Logic.

5. Van Der Waerden’s Theorem Multidim VDW theorem, upper and lower bounds on VDW numbers. APPLICATION to Number Theory, Communication Complexity, and the Diag-queens problem.

6. Roth’s Theorem for $k = 3$ (the combinatorial proof by Szemeredi).

7. Grid Colorings APPLICATION: A good example for lower bounds in Tree Resolution.

8. Rado’s theorem

9. Hales-Jewitt Theorem APPLICATION to communication complexity.

10. Polynomial VDW theorem APPLICATION to graph theory.