

Homework 3, Morally Due Tue Feb 19, 2013

COURSE WEBSITE: <http://www.cs.umd.edu/gasarch/858/S13.html>

(The symbol before gasarch is a tilde.)

- (0 points) What is your name? Write it clearly. Staple your HW. When is the midterm (give Date and Time)? If you cannot make it in that day/time see me ASAP. Join the Piazza group for the course. The codename is cm58. Look at the link on the class webpage about projects. Come see me about a project. READ the note on the class webpage that say THIS YOU SHOULD READ that you haven't already read.
- (50 points) Let  $(X, \preceq)$  be a set with an order on it. Let  $\preceq_{\text{awesme}}$  be the following order on  $X^*$

$$x_1x_2 \cdots x_n \preceq_{\text{awesme}} y_1y_2 \cdots y_m$$

if there exists  $i_1 < i_2 < \dots < i_n$  (numeric order) such that

$$x_1 \preceq y_{i_1}$$

$$x_2 \preceq y_{i_2}$$

⋮

$$x_n \preceq y_{i_n}.$$

Show that if  $(X, \preceq)$  is a wqo then  $(X^*, \preceq_{\text{awesme}})$  is a wqo. (HINT: This is similar to the proof that  $\Sigma^*$  under subsequence is a wqo.)

- (50 points) Find a function  $f(k)$  such that the following two statements are true:
  - For all colorings of  $[f(k)]$  either there are  $k$  numbers colored the same or there are  $k$  numbers colored differently.
  - There is a coloring of  $[f(k) - 1]$  such that there are NO  $k$  numbers colored the same, NOR are there  $k$  numbers colored differently.
- (Extra Credit- hand in to bill on sep sheet.) Let  $(X, \preceq)$  be a wqo. Let  $\binom{X}{<\omega}$  be the set of all FINITE subsets of  $X$ . We order  $\binom{X}{<\omega}$  by, if  $A, B \in \binom{X}{<\omega}$ , then  $A \preceq' B$  if there is a 1-1 map  $f$  from  $A$  to  $B$  where, for all  $x \in A$ ,  $x \preceq f(x)$ . Show that  $(\binom{X}{<\omega}, \preceq')$  is a wqo.